

Dynamic Allocation of Reusable Resources to Strategic Agents under Long-Term Constraints

(NeurIPS'25; **Winner** of ACM Student Research @ SIGMETRICS'25)

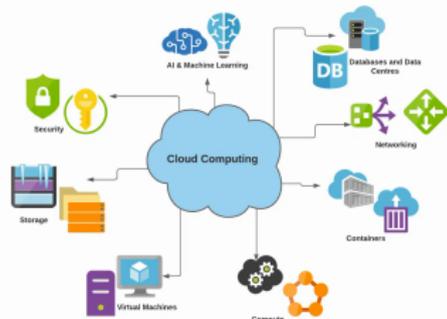
Yan Dai Negin Golrezaei Patrick Jaillet

Massachusetts Institute of Technology



Resource Allocation under Incentives & Constraints

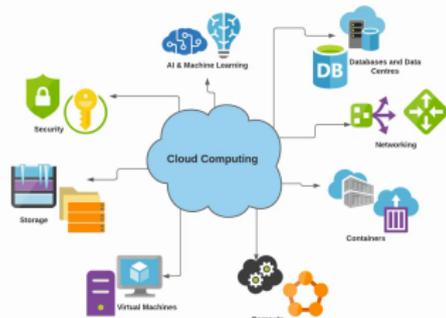
GPU Allocation



- **Resource:** Reusable GPU
- **Agents:** Research groups
- **Constr:** Energy & budget

Resource Allocation under Incentives & Constraints

GPU Allocation



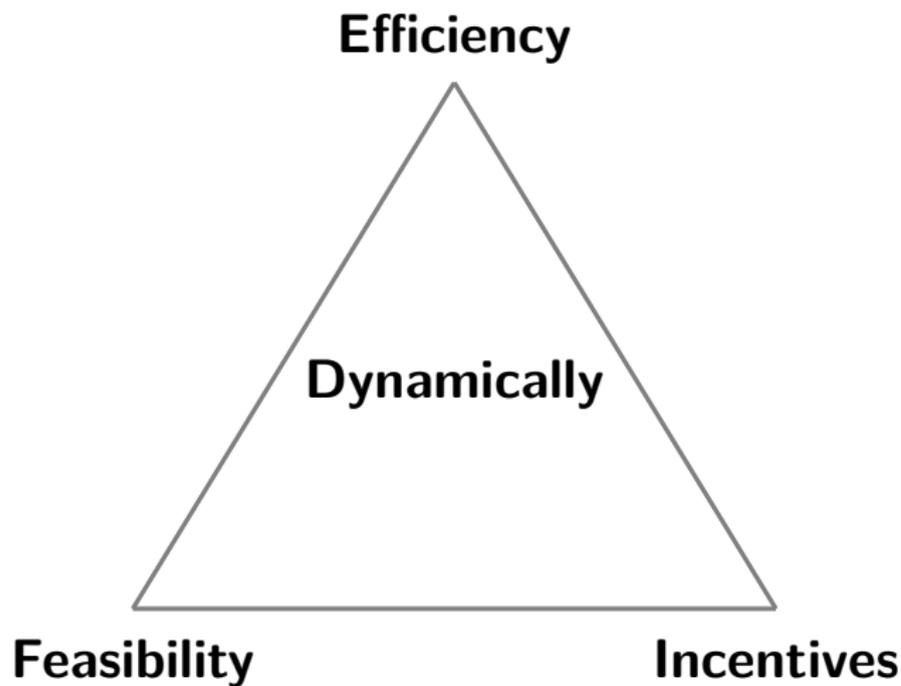
- **Resource:** Reusable GPU
- **Agents:** Research groups
- **Constr:** Energy & budget

Mobile Health Unit



- **Resource:** MHU
- **Agents:** Remote regions
- **Constr:** Staffing & budget

Efficiency-Feasibility-Incentives Trilemma



Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents,
value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

- Alloc cost $c_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t c_{t,i_t} \leq T\rho$$

Feasibility

Efficiency-Feasibility-Incentives Trilemma

Efficiency

Dynamically

Feasibility

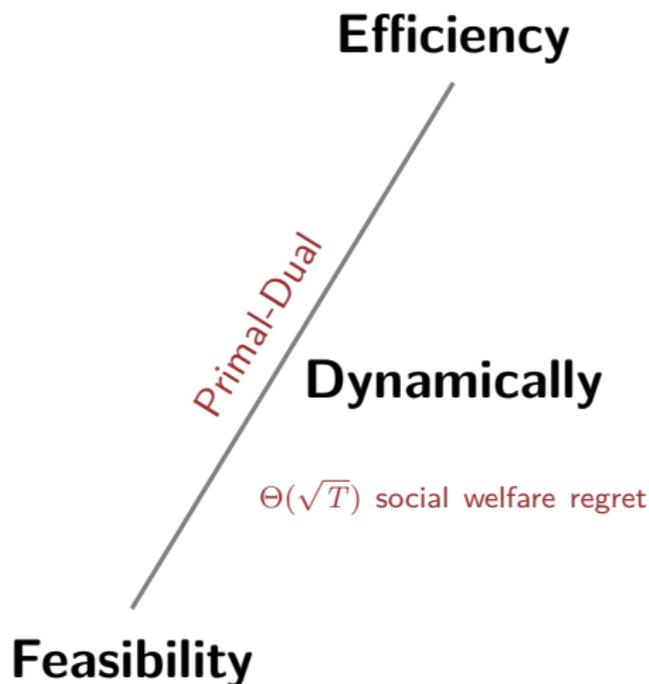
- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

- Alloc cost $c_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t c_{t,i_t} \leq T\rho$$

Efficiency-Feasibility-Incentives Trilemma



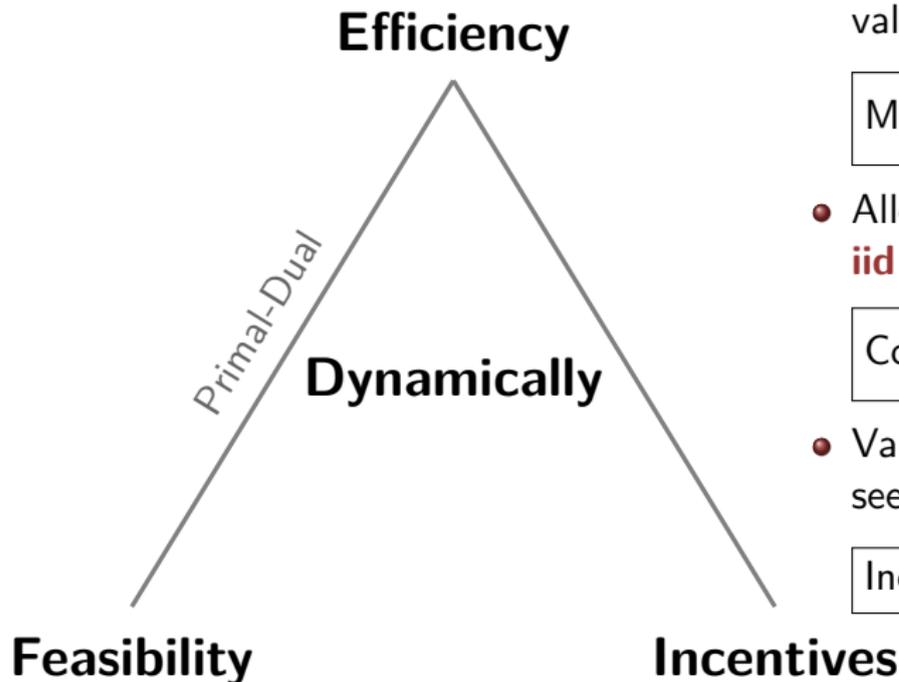
- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

- Alloc cost $c_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t c_{t,i_t} \leq T\rho$$

Efficiency-Feasibility-Incentives Trilemma



- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

- Alloc cost $\mathbf{c}_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t \mathbf{c}_{t,i_t} \leq T\rho$$

- Value $v_{t,i}$ private; only see **strategic report** $u_{t,i}$

$$\text{Incentivize } u_{t,i} \approx v_{t,i}$$

Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents,
value $v_{t,i} \sim$ **unknown** v_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

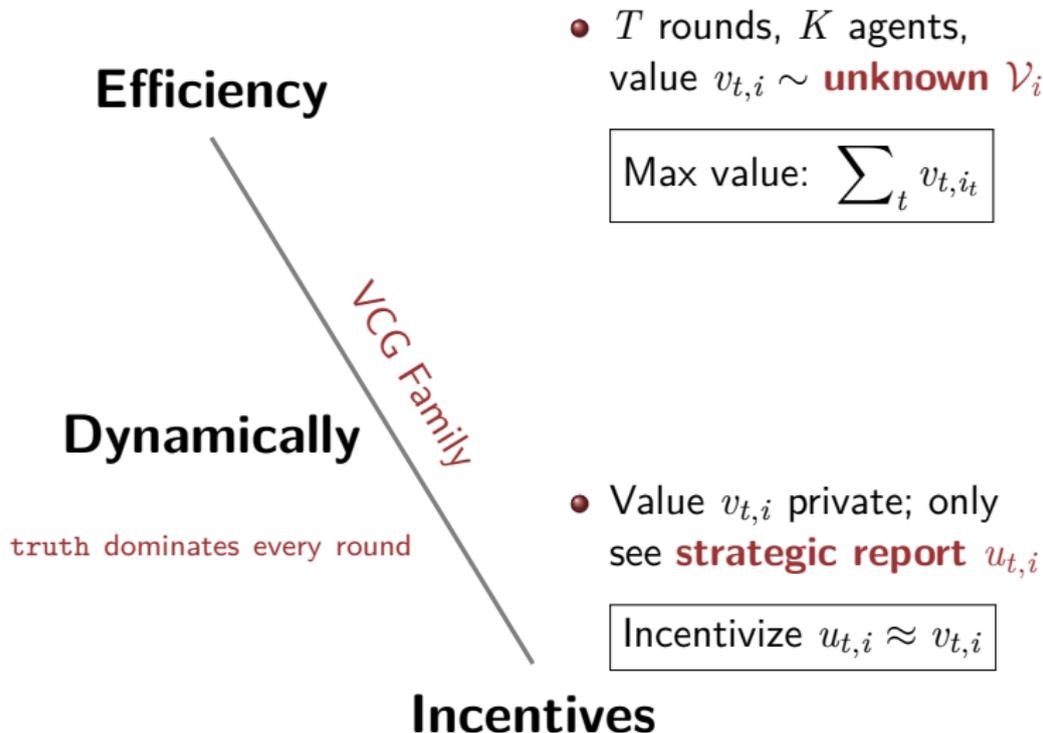
Dynamically

- Value $v_{t,i}$ private; only
see **strategic report** $u_{t,i}$

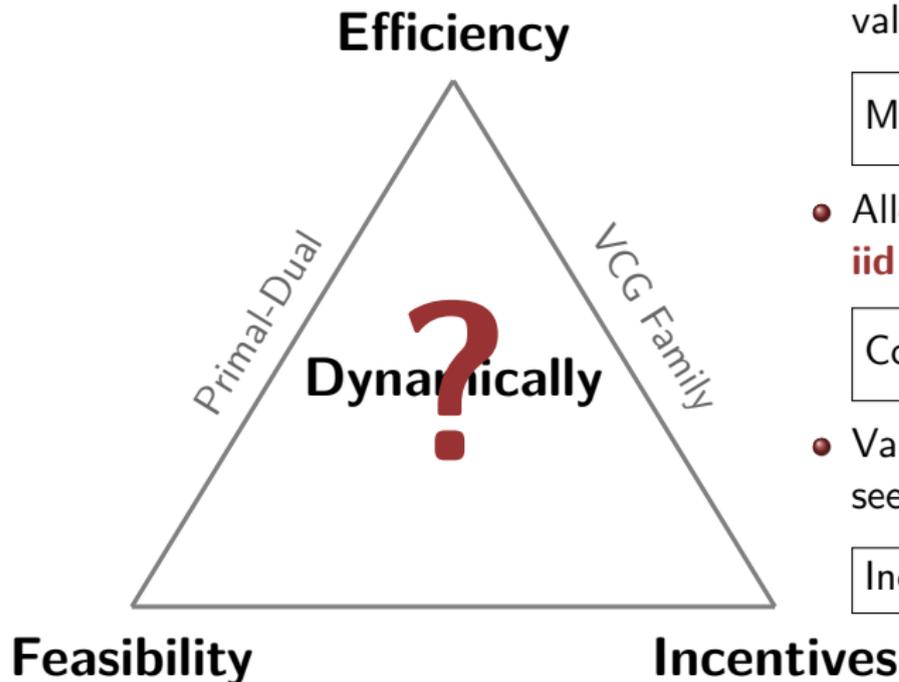
$$\text{Incentivize } u_{t,i} \approx v_{t,i}$$

Incentives

Efficiency-Feasibility-Incentives Trilemma



Efficiency-Feasibility-Incentives Trilemma



- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

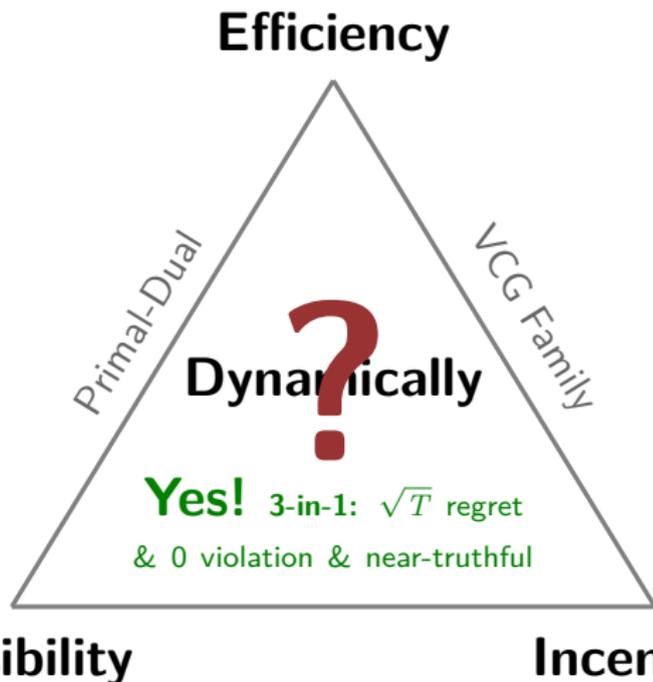
- Alloc cost $\mathbf{c}_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t \mathbf{c}_{t,i_t} \leq T\rho$$

- Value $v_{t,i}$ private; only see **strategic report** $u_{t,i}$

$$\text{Incentivize } u_{t,i} \approx v_{t,i}$$

Efficiency-Feasibility-Incentives Trilemma



- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

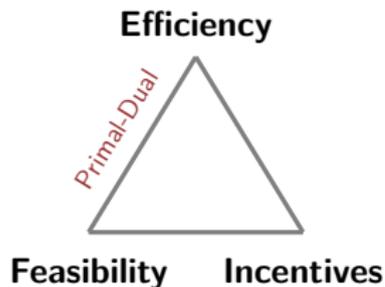
- Alloc cost $\mathbf{c}_{t,i}$ (d -dim), **iid** \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t \mathbf{c}_{t,i_t} \leq T\rho$$

- Value $v_{t,i}$ private; only see **strategic report** $u_{t,i}$

$$\text{Incentivize } u_{t,i} \approx v_{t,i}$$

Classical Primal-Dual Fails with Strategic Agents



Primal (Good Allocations)

Dual (Track Constraints)

Figure: Classical Primal-Dual in Round t

Classical Primal-Dual Fails with Strategic Agents

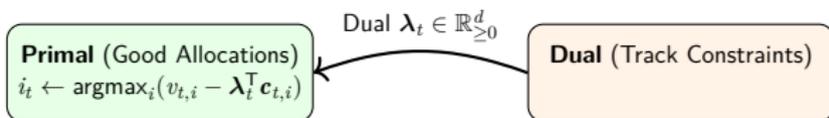
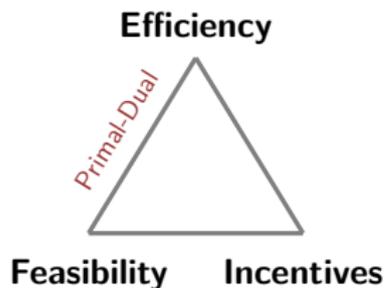


Figure: Classical Primal-Dual in Round t

Classical Primal-Dual Fails with Strategic Agents

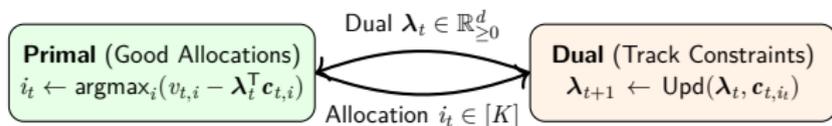
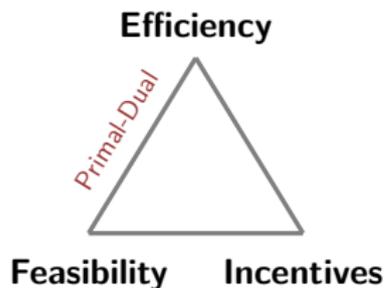


Figure: Classical Primal-Dual in Round t

Classical Primal-Dual Fails with Strategic Agents

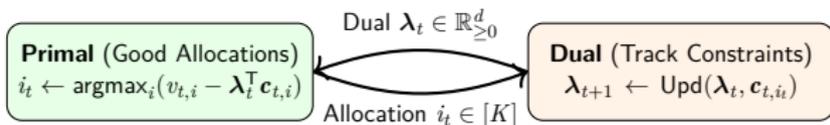
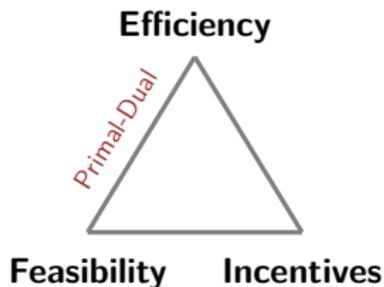


Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?

Manipulate $v_{t,i}$



Alter i_t

Classical Primal-Dual Fails with Strategic Agents

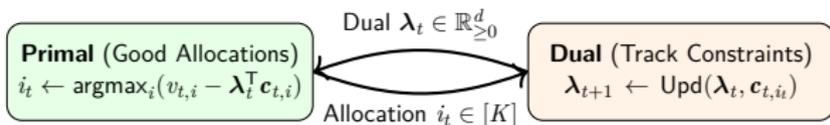
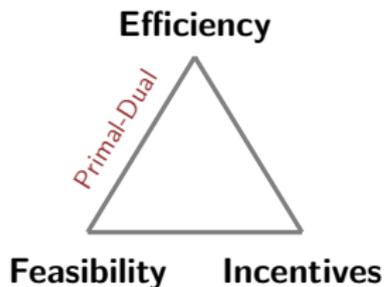
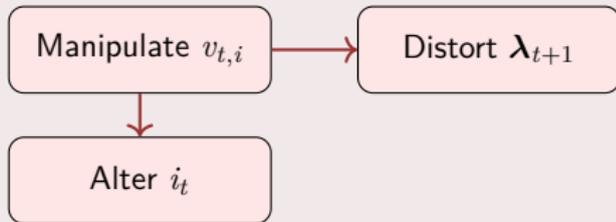


Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?



Classical Primal-Dual Fails with Strategic Agents

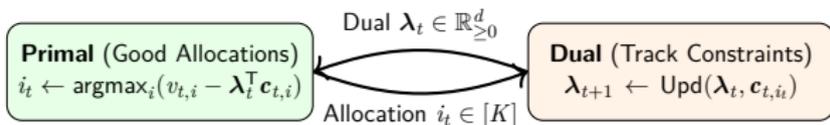
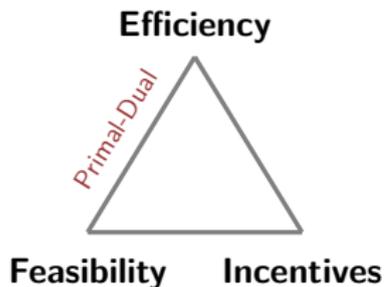
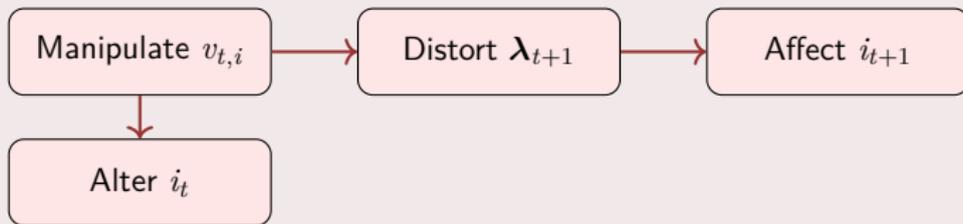


Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?



Classical Primal-Dual Fails with Strategic Agents

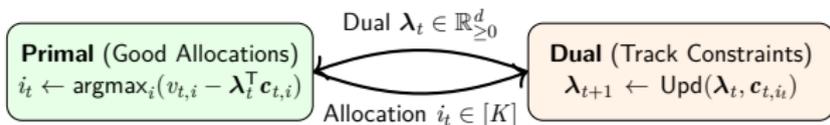
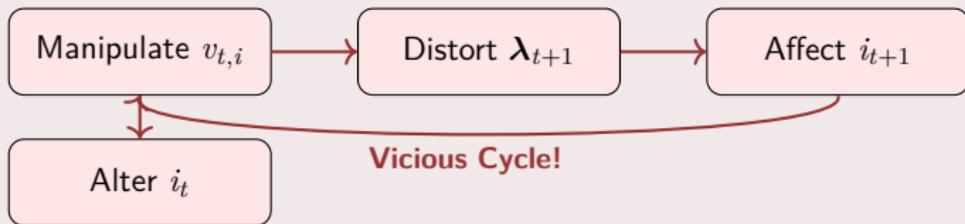


Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?



Classical Primal-Dual Fails with Strategic Agents

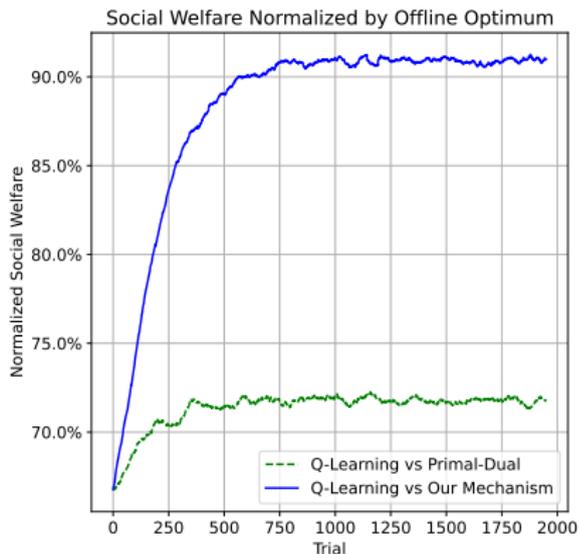
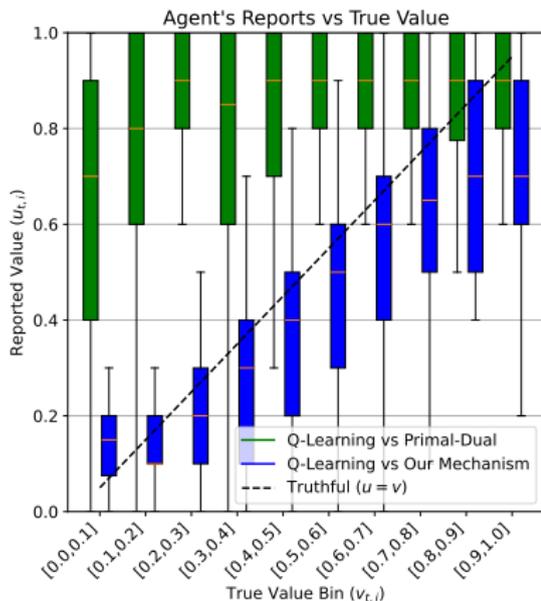


Figure: Vanilla Primal-Dual vs Our Mechanism

Classical Primal-Dual Fails with Strategic Agents

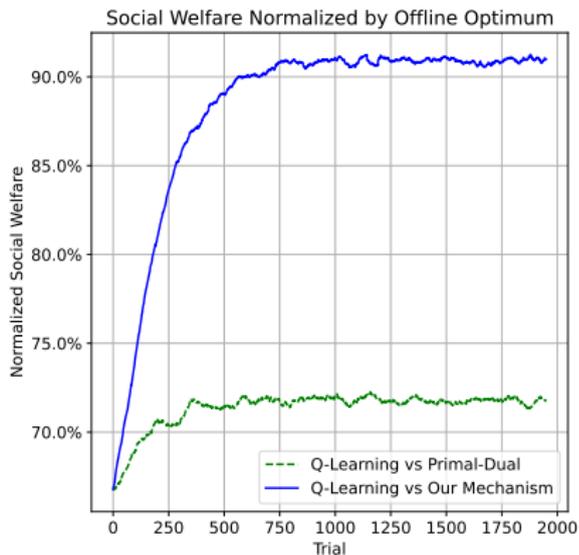
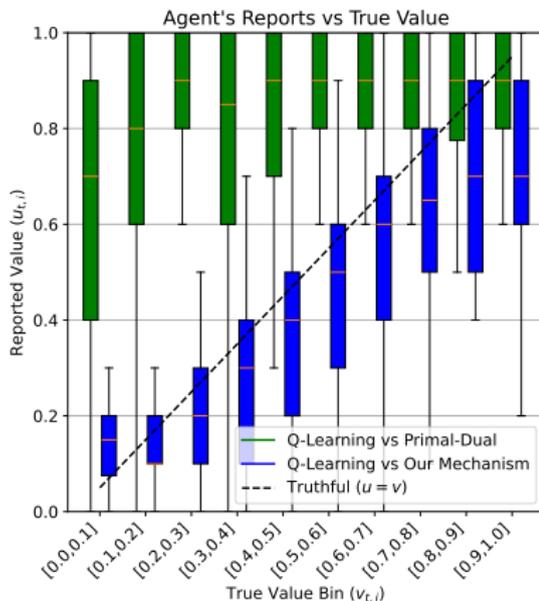


Figure: Vanilla Primal-Dual vs Our Mechanism

misreport vs truthful

Classical Primal-Dual Fails with Strategic Agents

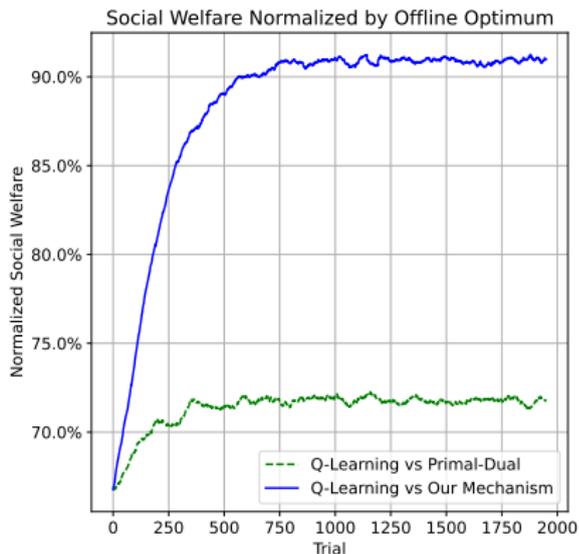
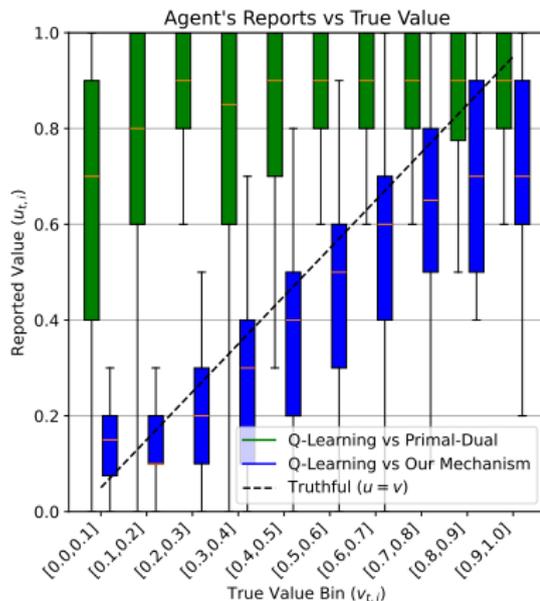
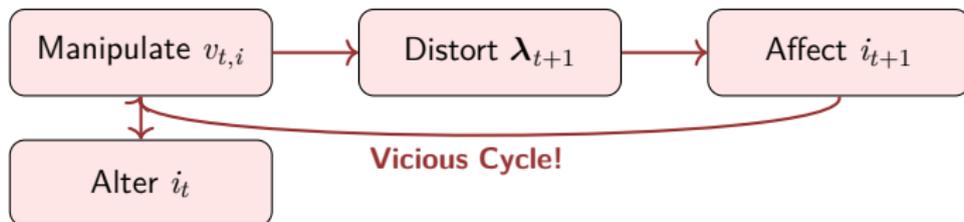


Figure: Vanilla Primal-Dual vs Our Mechanism

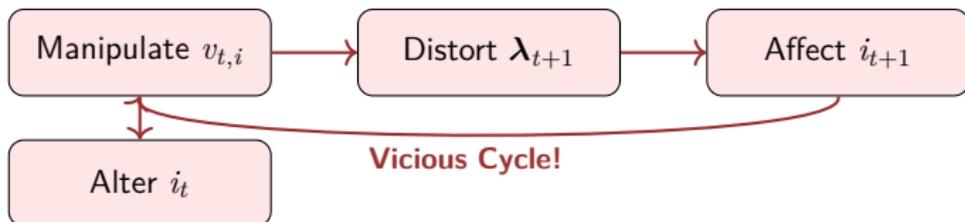
misreport vs truthful

low vs high efficiency

Primal Allocations

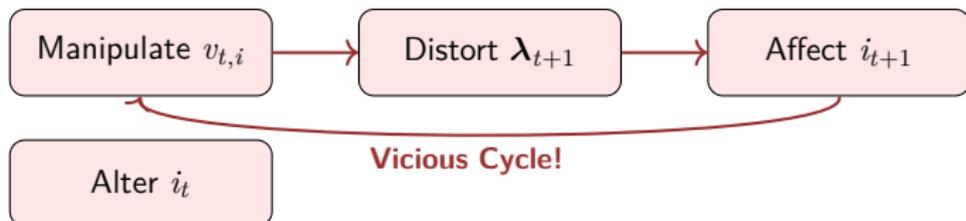


Primal Allocations: Pricing



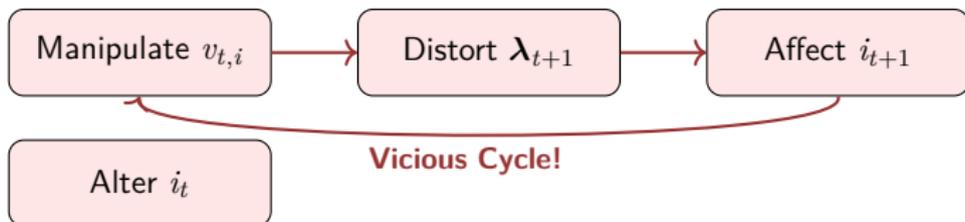
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)

Primal Allocations: Pricing



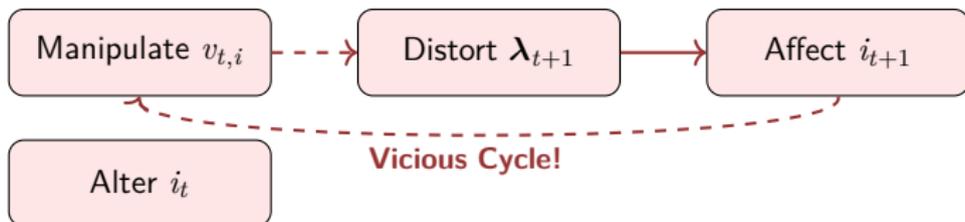
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
 \implies truth dominates (for static setups)

Primal Allocations: Pricing + Epoching



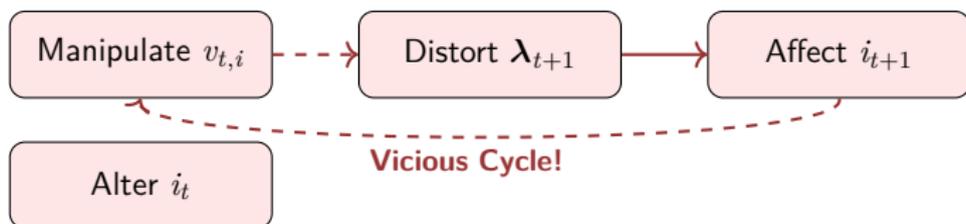
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
 \implies truth dominates (for static setups)
- 2 **Epoch-Based Lazy Updates.** Fix λ for \sqrt{T} rounds

Primal Allocations: Pricing + Epoching



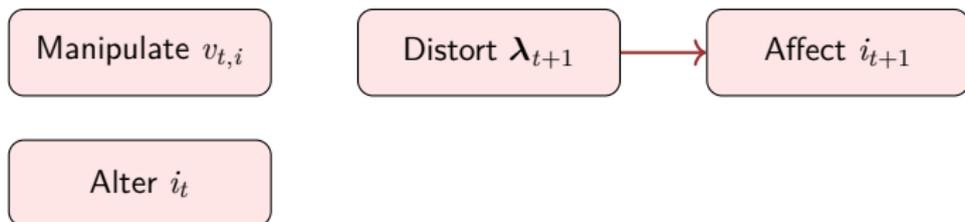
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
 \implies truth dominates (for static setups)
- 2 **Epoch-Based Lazy Updates.** Fix λ for \sqrt{T} rounds
 \implies hard to affect future (but not impossible)

Primal Allocations: Pricing + Epoching + Exploration



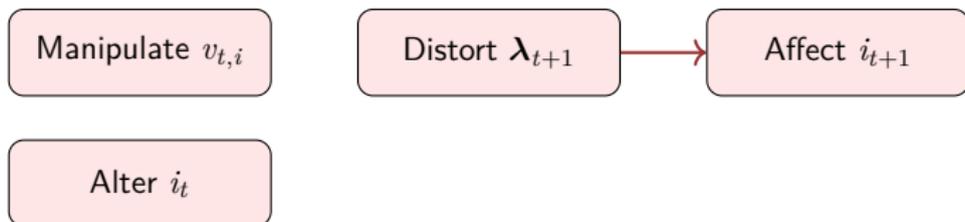
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
 \implies truth dominates (for static setups)
- 2 **Epoch-Based Lazy Updates.** Fix λ for \sqrt{T} rounds
 \implies hard to affect future (but not impossible)
- 3 **Randomized Exploration.** Misreport means harm

Primal Allocations: Pricing + Epoching + Exploration



- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
 \implies truth dominates (for static setups)
- 2 **Epoch-Based Lazy Updates.** Fix λ for \sqrt{T} rounds
 \implies hard to affect future (but not impossible)
- 3 **Randomized Exploration.** Misreport means harm
 $\implies \exists$ near-truthful equilibrium (if harm \geq gain)

Primal Allocations: Pricing + Epoching + Exploration



- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)
⇒ truth dominates (for static setups)
- 2 **Epoch-Based Lazy Updates.** Fix λ for \sqrt{T} rounds
⇒ hard to affect future (but not impossible)
- 3 **Randomized Exploration.** Misreport means harm
⇒ \exists near-truthful equilibrium (if harm \geq gain)

Theorem. $\tilde{O}(1)$ misreports & $\tilde{O}(1)$ misallocations per epoch

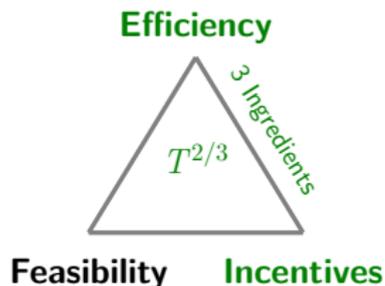
Dual Updates



Dual Updates



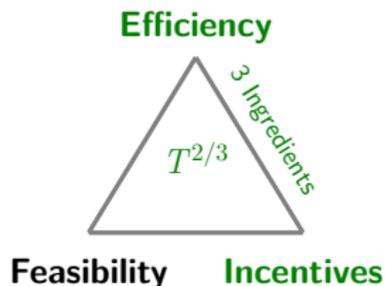
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

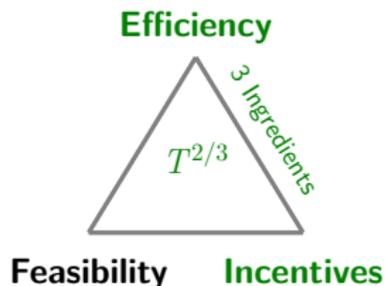
Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

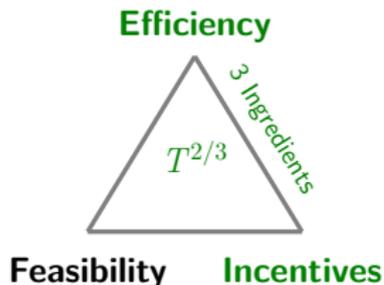
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

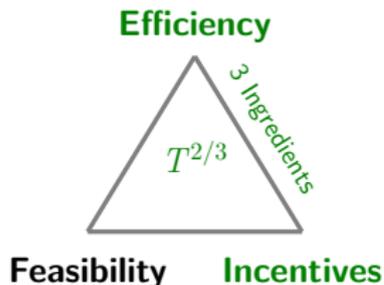
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual**

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

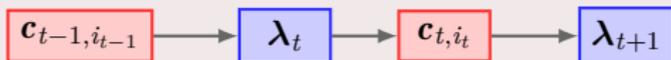
Theorem 1: Sublinear Regret ✓

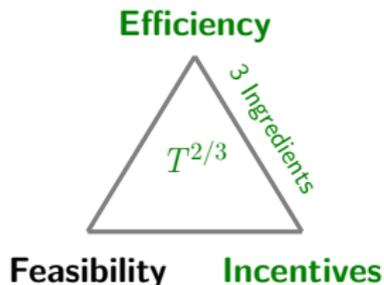
3 ingredients (primal) + GD / FTRL (dual)
 $\Rightarrow \tilde{O}(T^{2/3})$ **regret** (“no-regret” guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual**

No Lazy



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

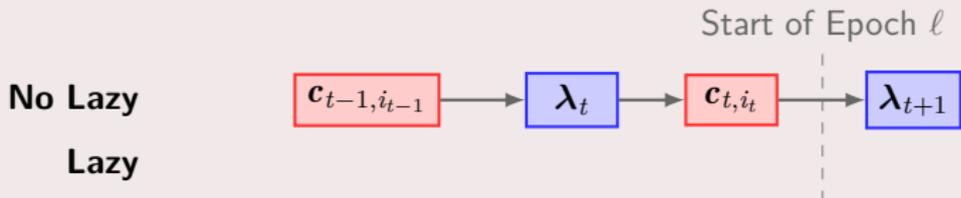
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

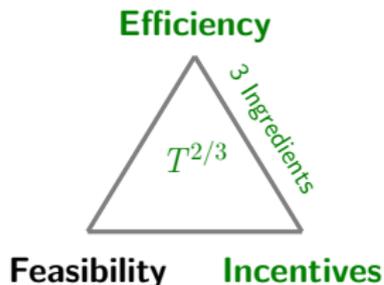
Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\Rightarrow \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual**



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

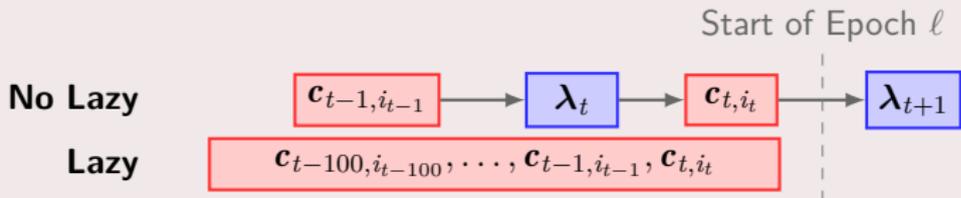
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

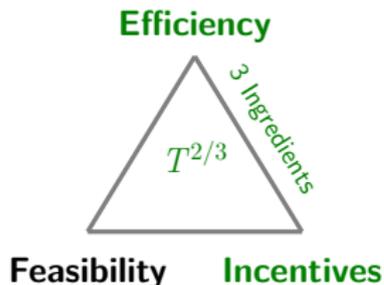
Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual**



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

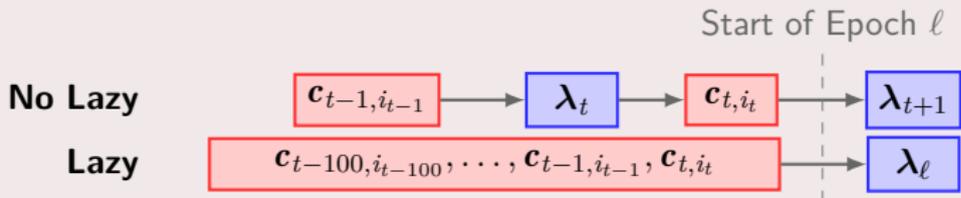
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

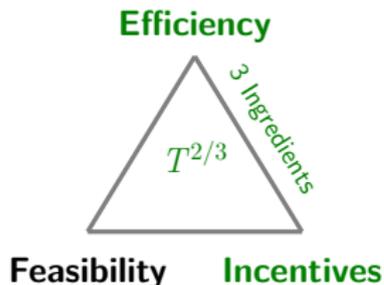
Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual**



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

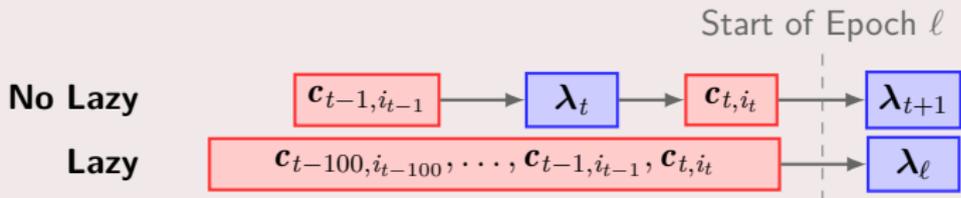
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

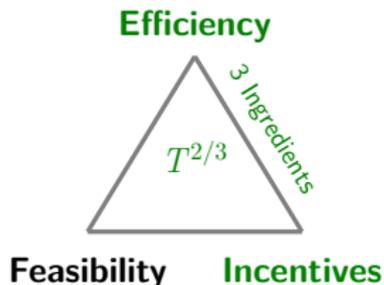
Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\Rightarrow \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual** (cannot react to c_{t,i_t} promptly)



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$ 

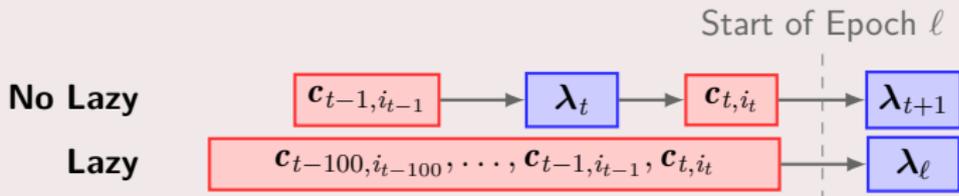
Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\implies \tilde{O}(T^{2/3})$ **regret** (“no-regret” guarantee)

Can we do better?

- Lazy updates **good for primal** (less incentives & abilities)
- Lazy updates **bad for dual** (cannot react to c_{t,i_t} promptly)



Theorem. “Low-switching online learning” has $\Omega(T^{2/3})$ **regret**

Dual Updates: Even Better via Predictability



Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- 1 Truthful \implies iid future cost \mathbf{c}_{t,i_t} ($i_t \approx \operatorname{argmax}_i (v_{t,i} - \lambda_\ell^\top \mathbf{c}_{t,i})$)

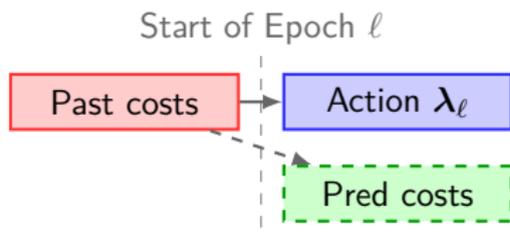
Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- ① Truthful \implies **iid future cost** \mathbf{c}_{t,i_t} ($i_t \approx \operatorname{argmax}_i (v_{t,i} - \lambda_\ell^\top \mathbf{c}_{t,i})$)
- ② Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)

Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- 1 Truthful \implies **iid future cost** \mathbf{c}_{t,i_t} ($i_t \approx \operatorname{argmax}_i (v_{t,i} - \boldsymbol{\lambda}_\ell^\top \mathbf{c}_{t,i})$)
 - 2 Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)
- \implies **Predict new costs**

Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- ① Truthful \implies **iid future cost** \mathbf{c}_{t,i_t} ($i_t \approx \operatorname{argmax}_i (v_{t,i} - \lambda_\ell^\top \mathbf{c}_{t,i})$)
 - ② Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)
- \implies **Predict new costs** for better action λ_ℓ

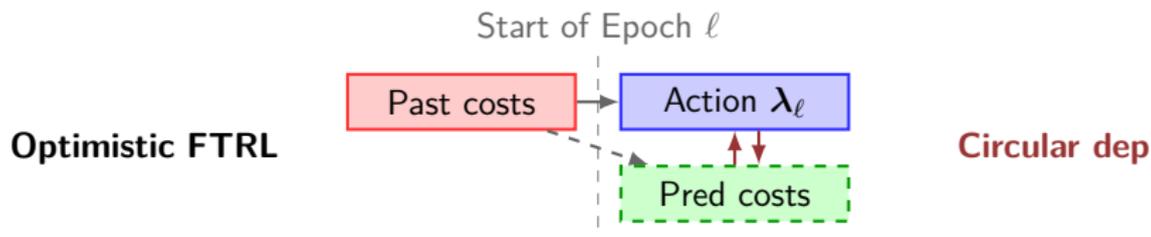
Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**

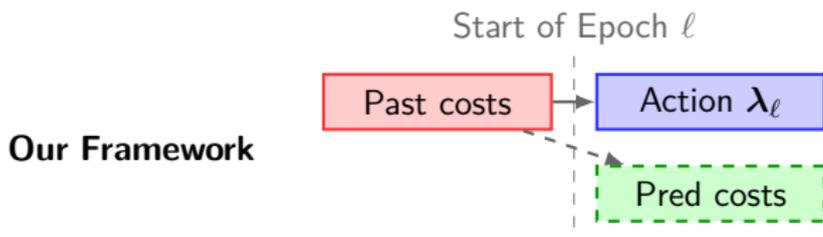
Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Novel online learning

Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Novel online learning

Dual Updates: Even Better via Predictability

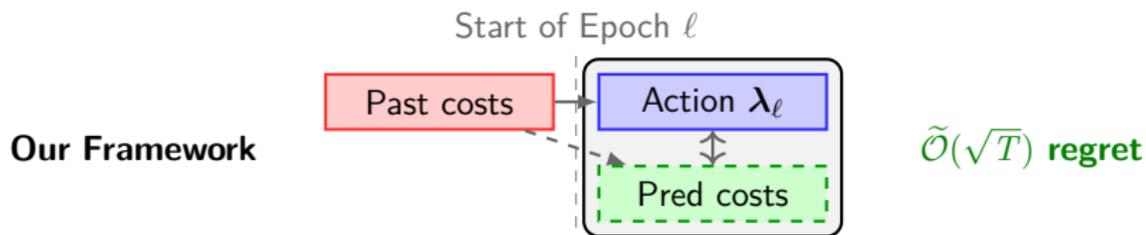


Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Novel online learning (decide action λ_ℓ & pred costs simultaneously via fixed-point subroutine; named **O-FTRL-FP**)

Dual Updates: Even Better via Predictability

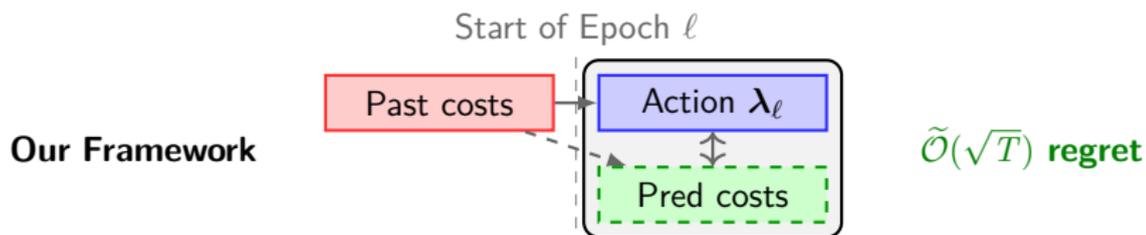


Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Novel online learning (decide action λ_ℓ & pred costs simultaneously via fixed-point subroutine; named **O-FTRL-FP**) $\implies \tilde{O}(\sqrt{T})$ **regret**

Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**
- Yield pred costs **as-if true dual = λ_ℓ**

Novel online learning (decide action λ_ℓ & pred costs simultaneously via fixed-point subroutine; named **O-FTRL-FP**) $\implies \tilde{O}(\sqrt{T})$ **regret**

Recall. Non-strategic lower bound = $\Omega(\sqrt{T})$ regret

Main Results & Takeaway

Main Result

1st dynamic mechanism resolving **trilemma**:

- **Efficiency.** Optimal $\tilde{O}(\sqrt{T})$ regret
- **Feasibility.** Zero constraint violation
- **Incentives.** Robust to strategic agents



Key Techniques

- **Primal Side: Incentive-Aware Allocation.** Novel mixture of dual-adjusted pricing + lazy updates + random exploration
- **Dual Side: Online Learning for Updates.** Truthfulness \Rightarrow Predictability + novel framework for circular dependencies

Questions are more than welcomed!

✉ yandai20@mit.edu; 🌐 <https://yandaichn.github.io/>