清華大学 交叉信息研究院
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Banker Online Mirror Descent

— A Universal Approach for Delayed Online Bandit Learning

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[*]: Equal contribution.

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- **Optimal regret achieved by Zimmert et al. (2020):**

$$O\left(\sqrt{KT} + \sqrt{D \log K}\right).$$

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- Want **a universal approach** to handle delays robustly!

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- Sadly, vanilla OMD cannot handle delays

Vanilla OMD

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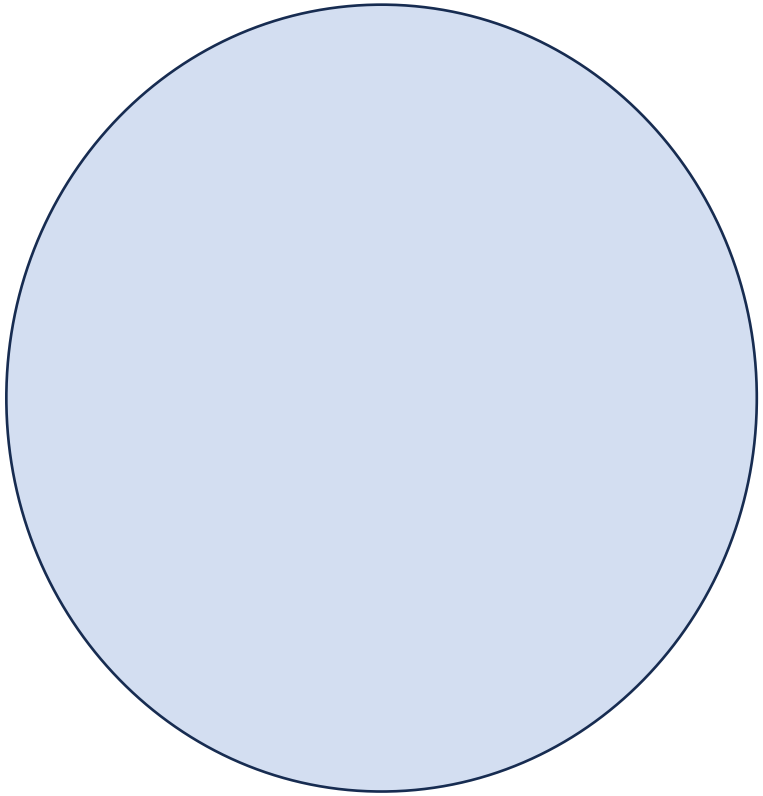
Primal Space Δ^K



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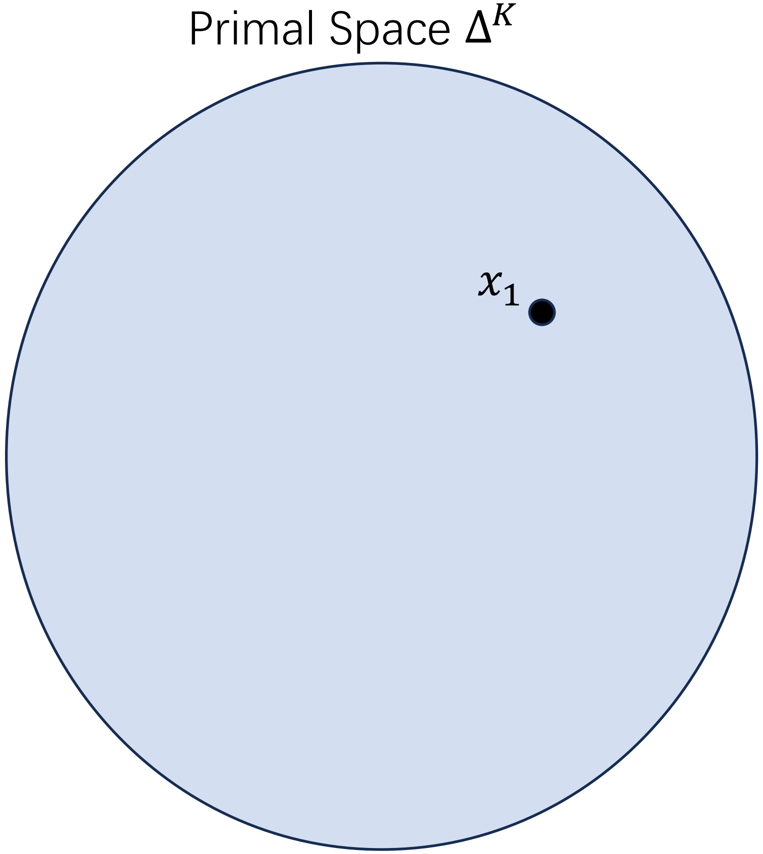

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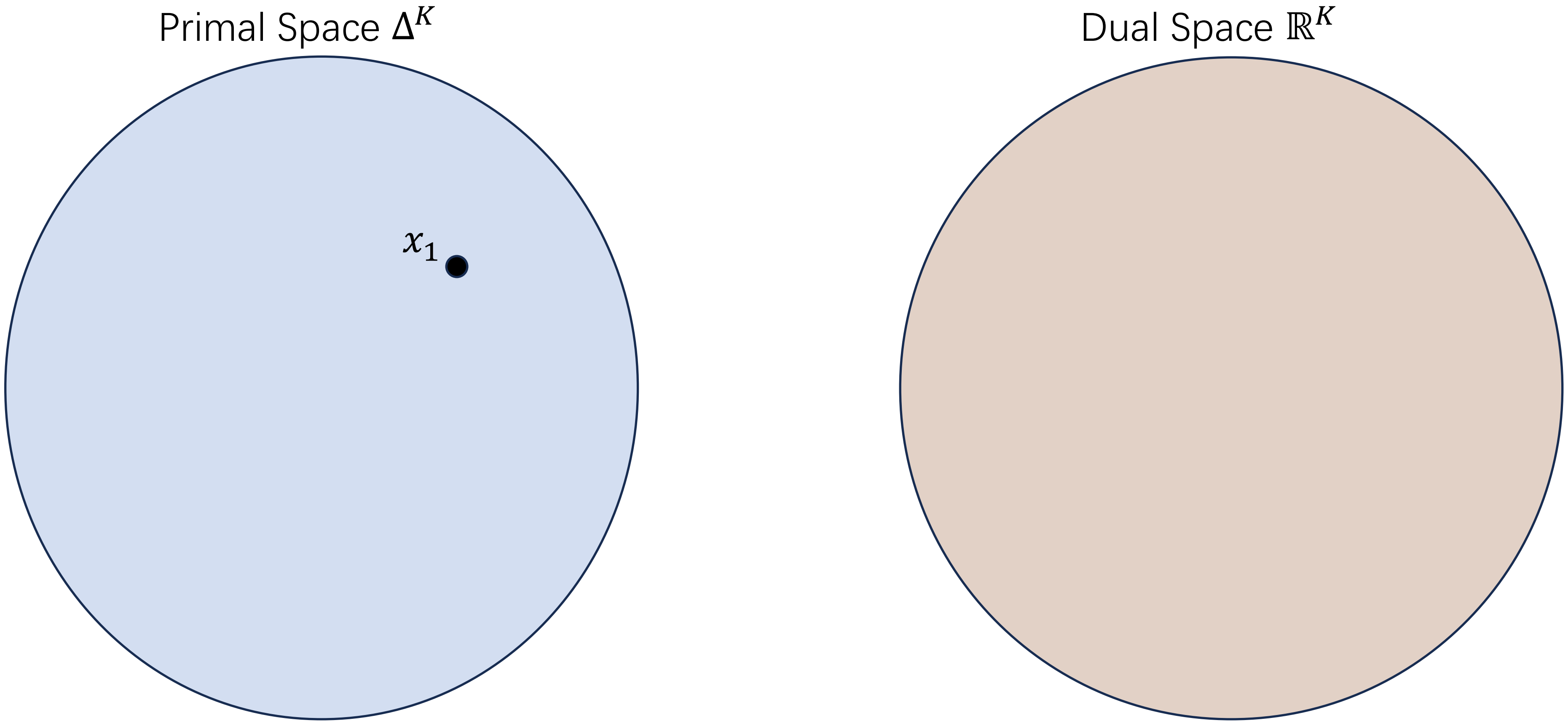

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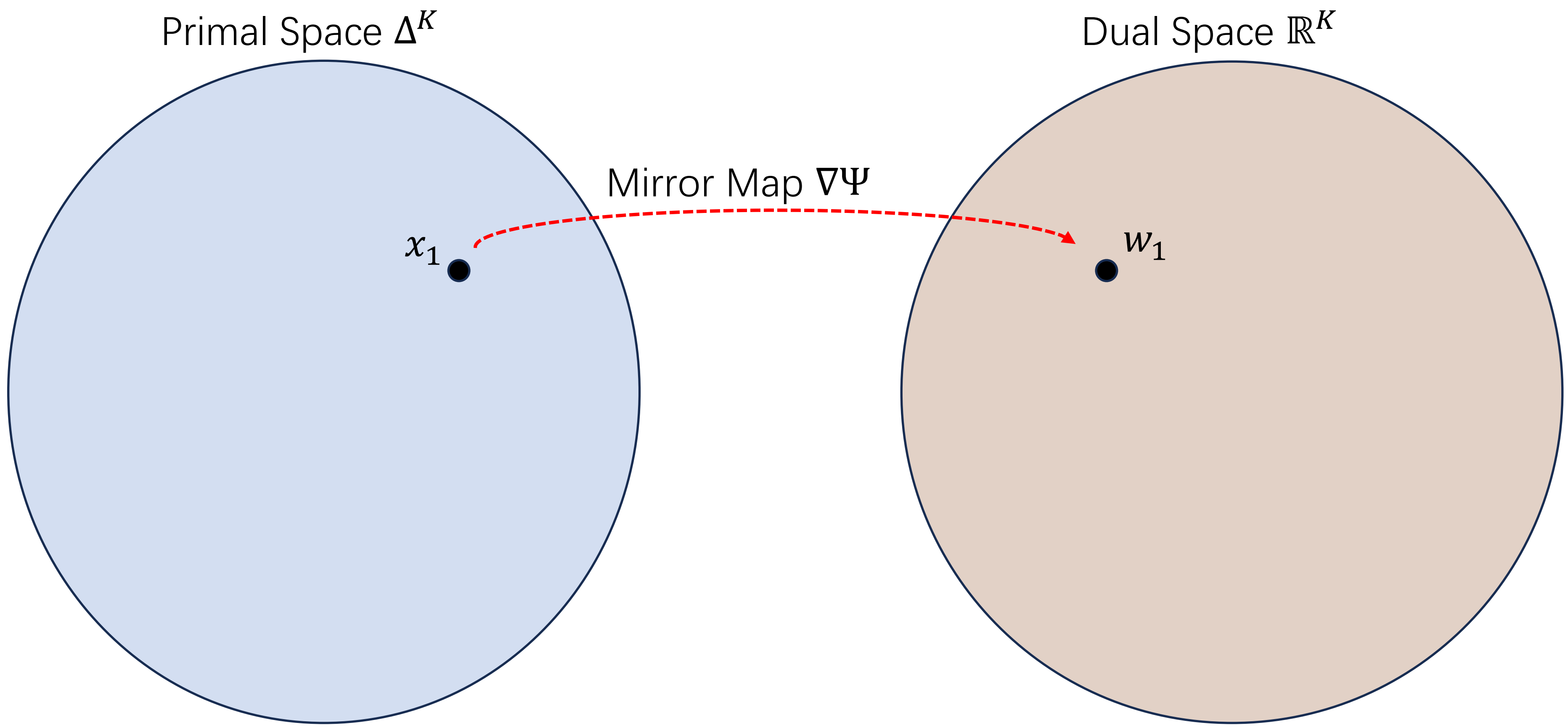

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Dual Space $\mathbb{R}^K$
Mirror Map $\nabla\Psi$
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w_1

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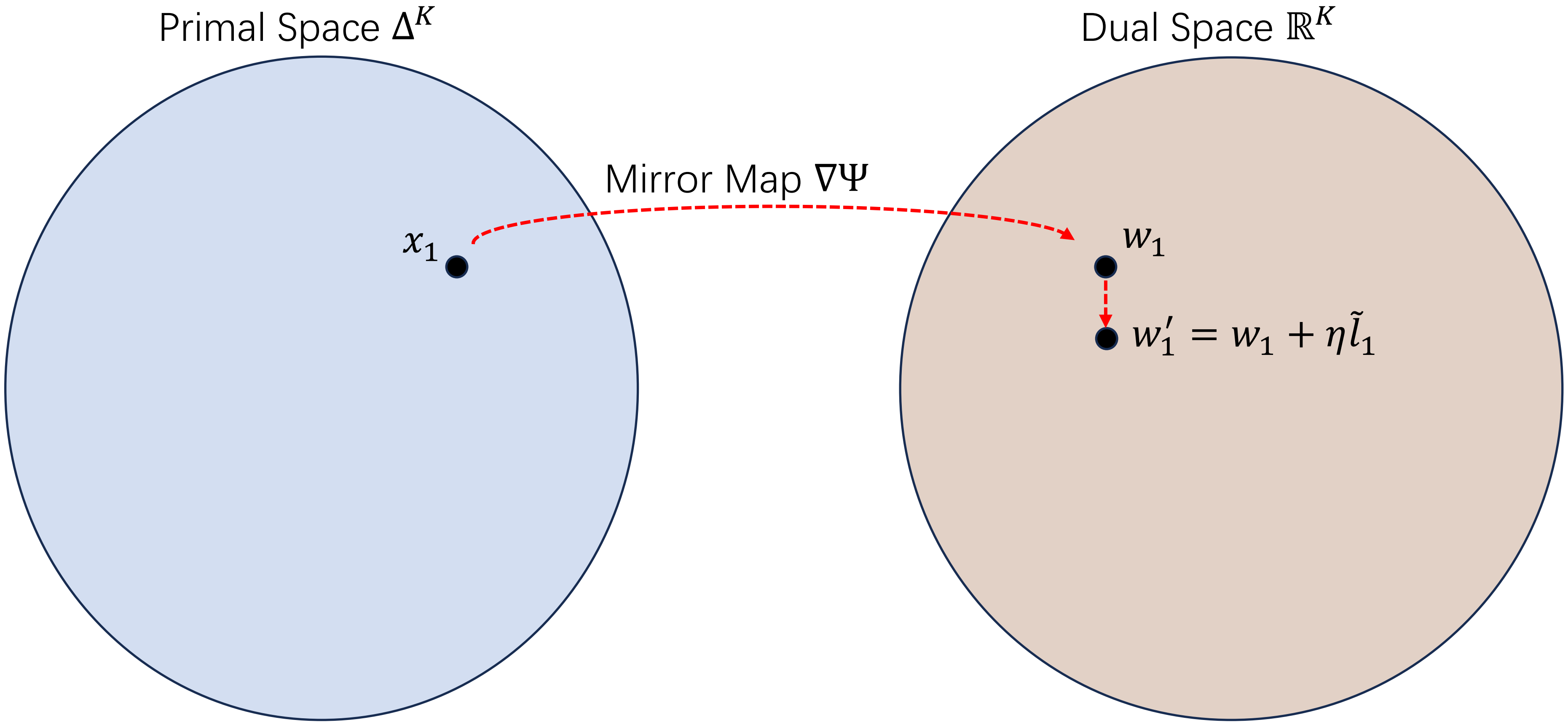

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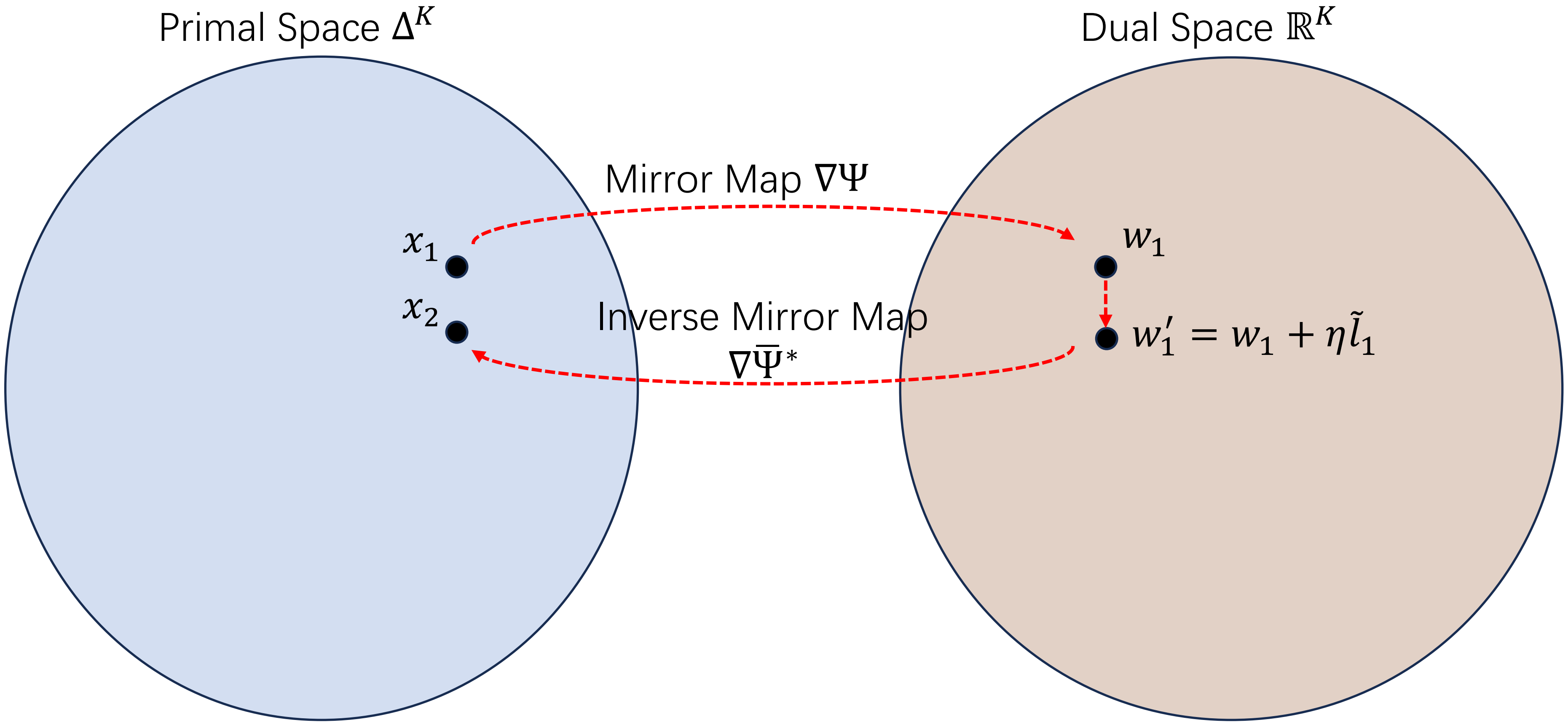

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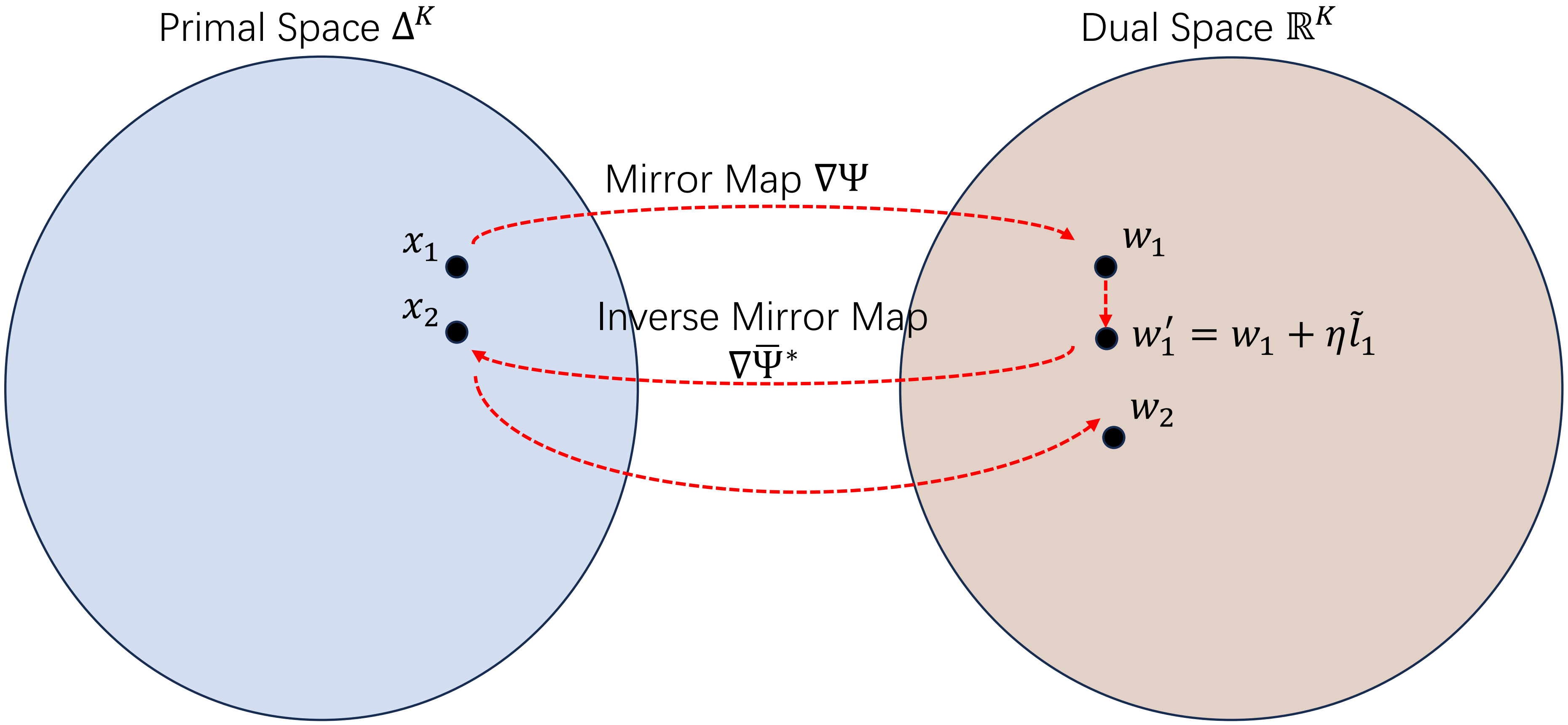

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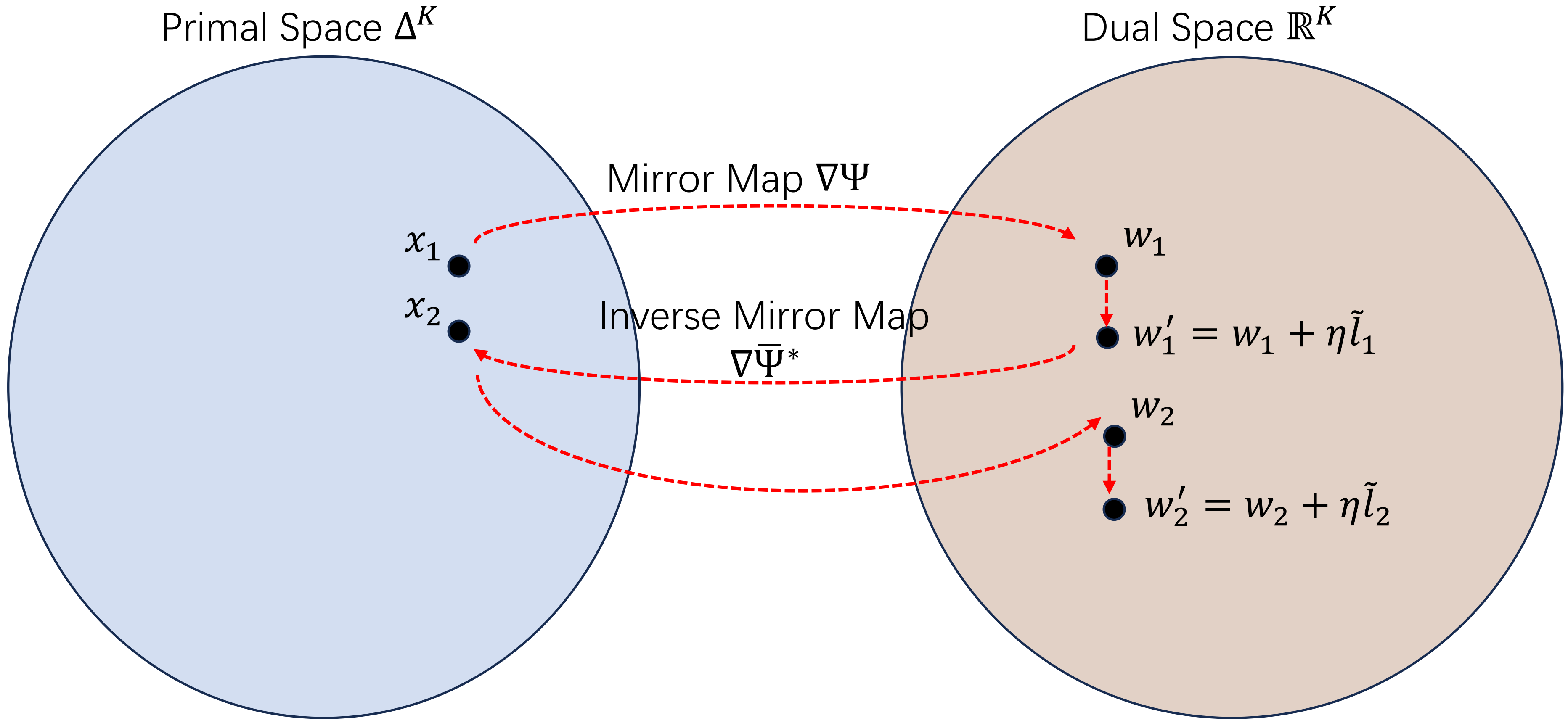

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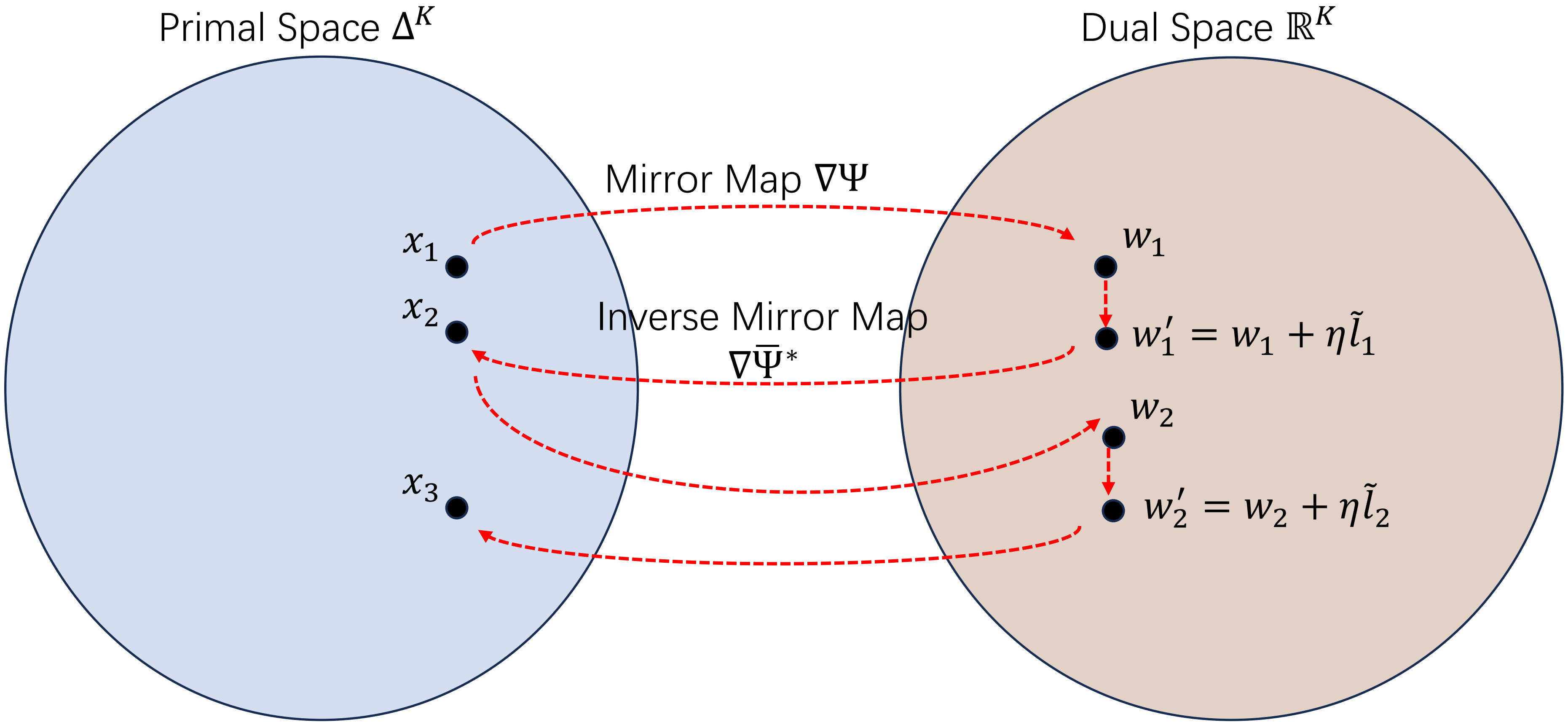
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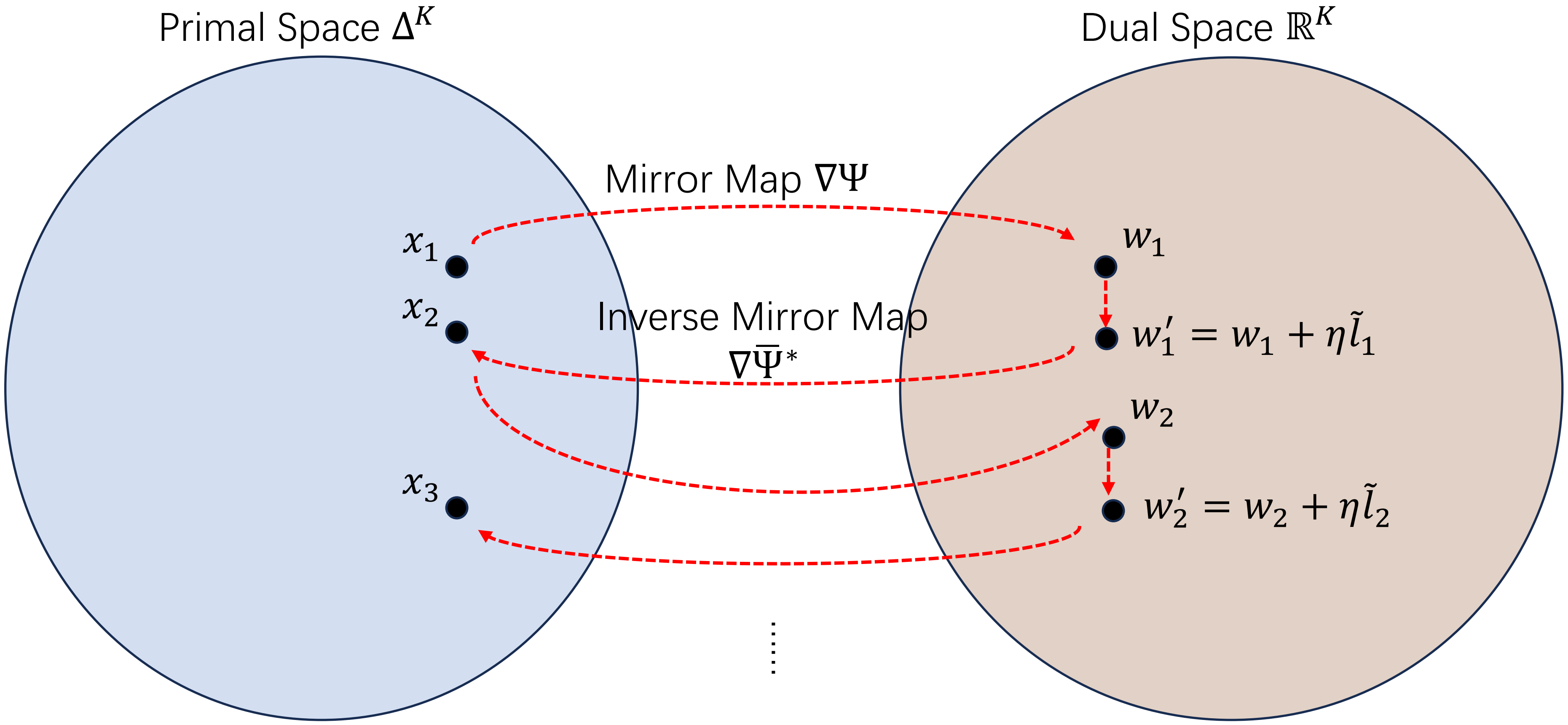

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- Single-step OMD lemma:

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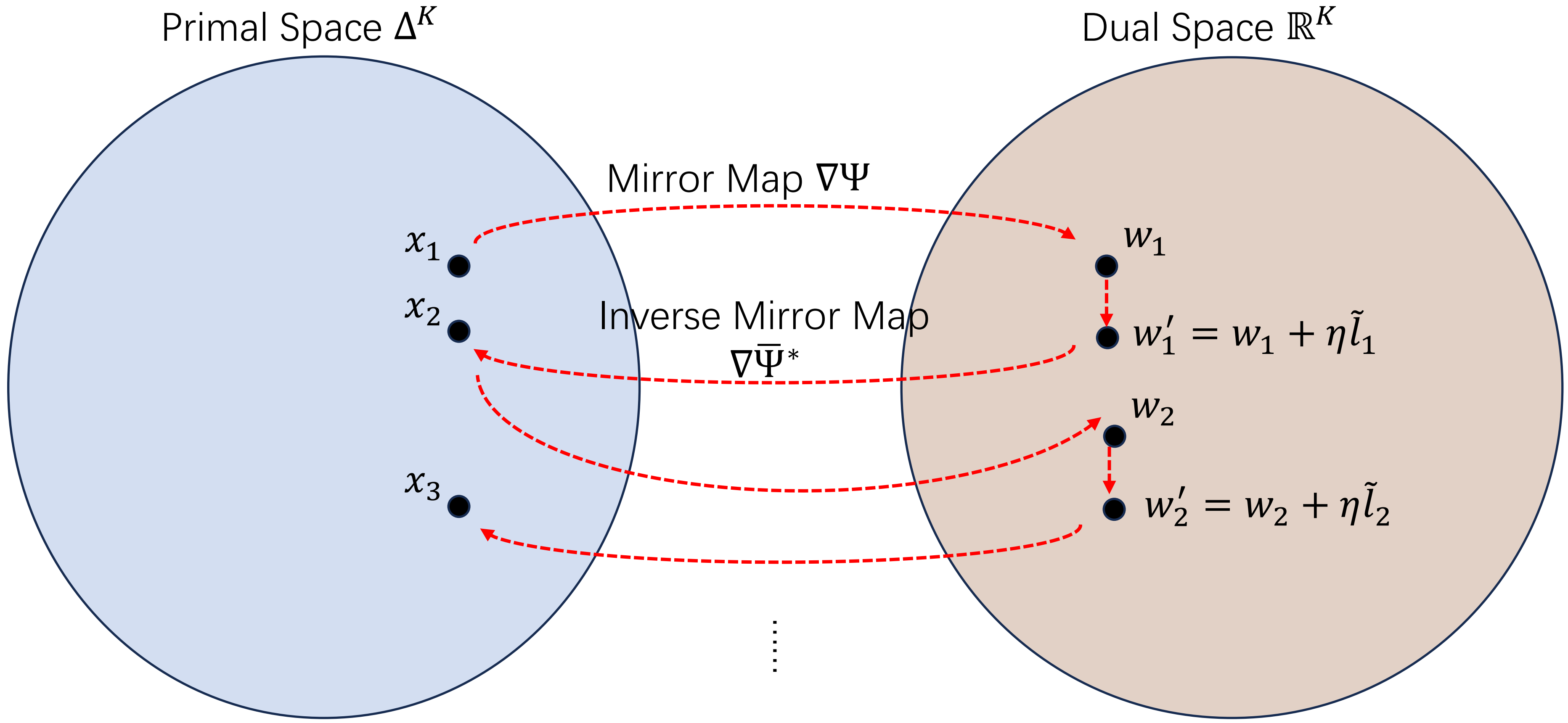

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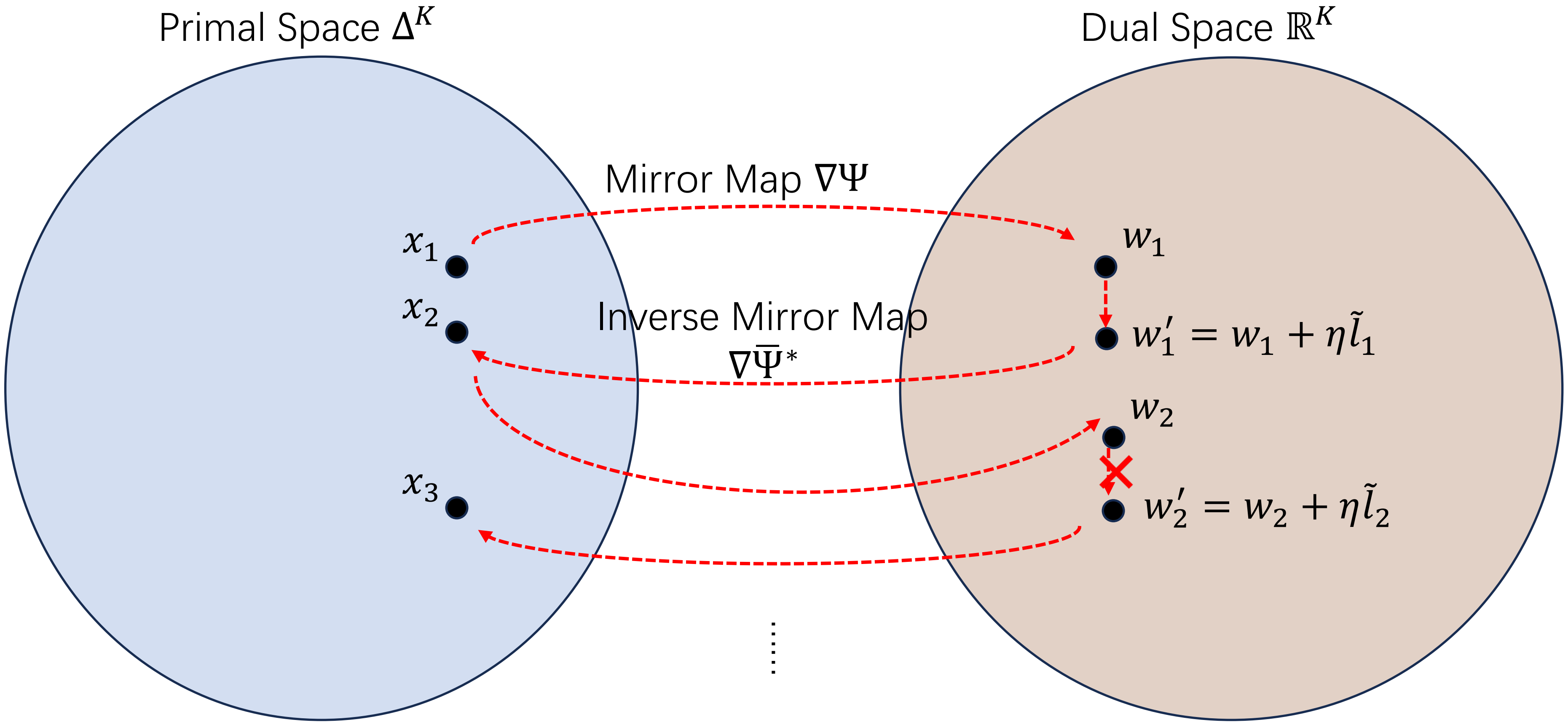

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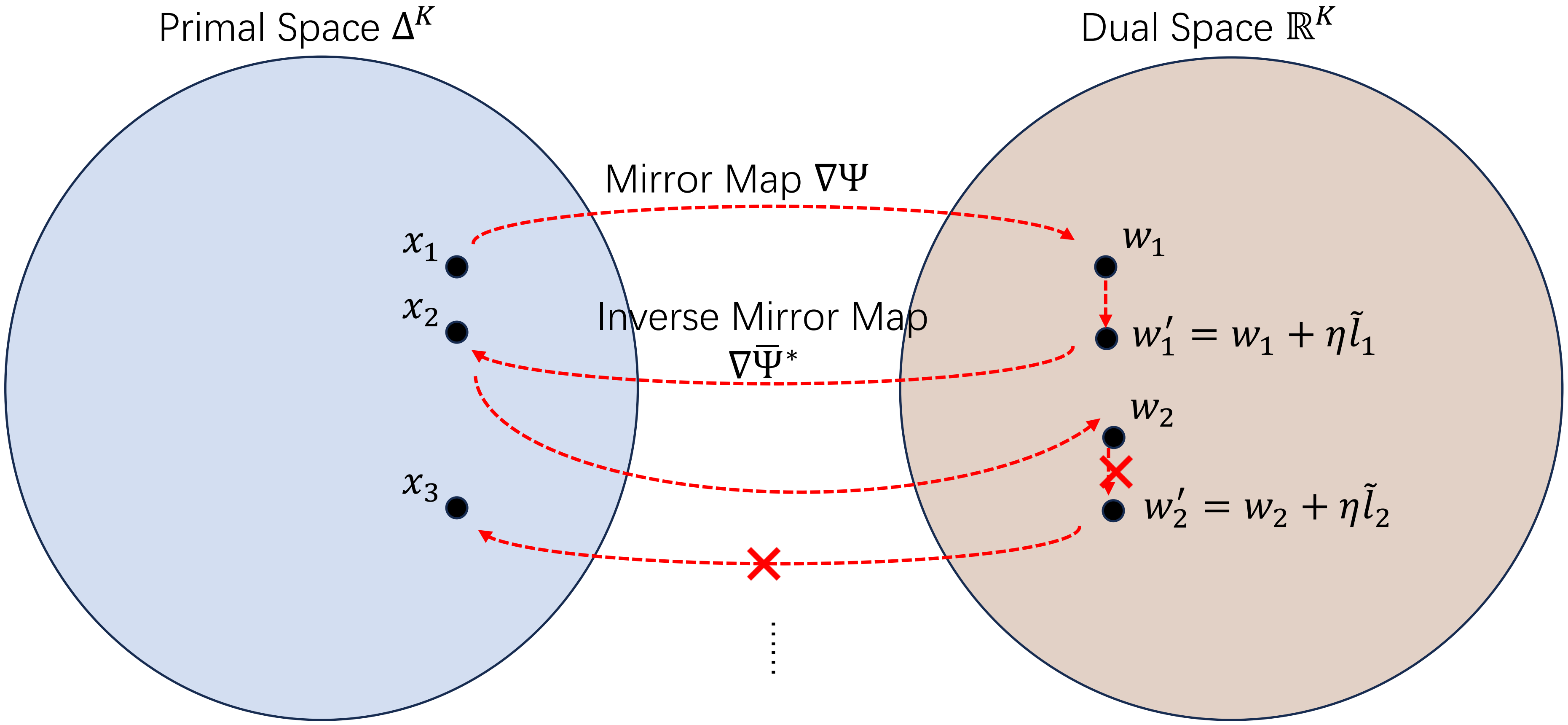

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  - Fine-grained analysis of potential terms due to OMD steps

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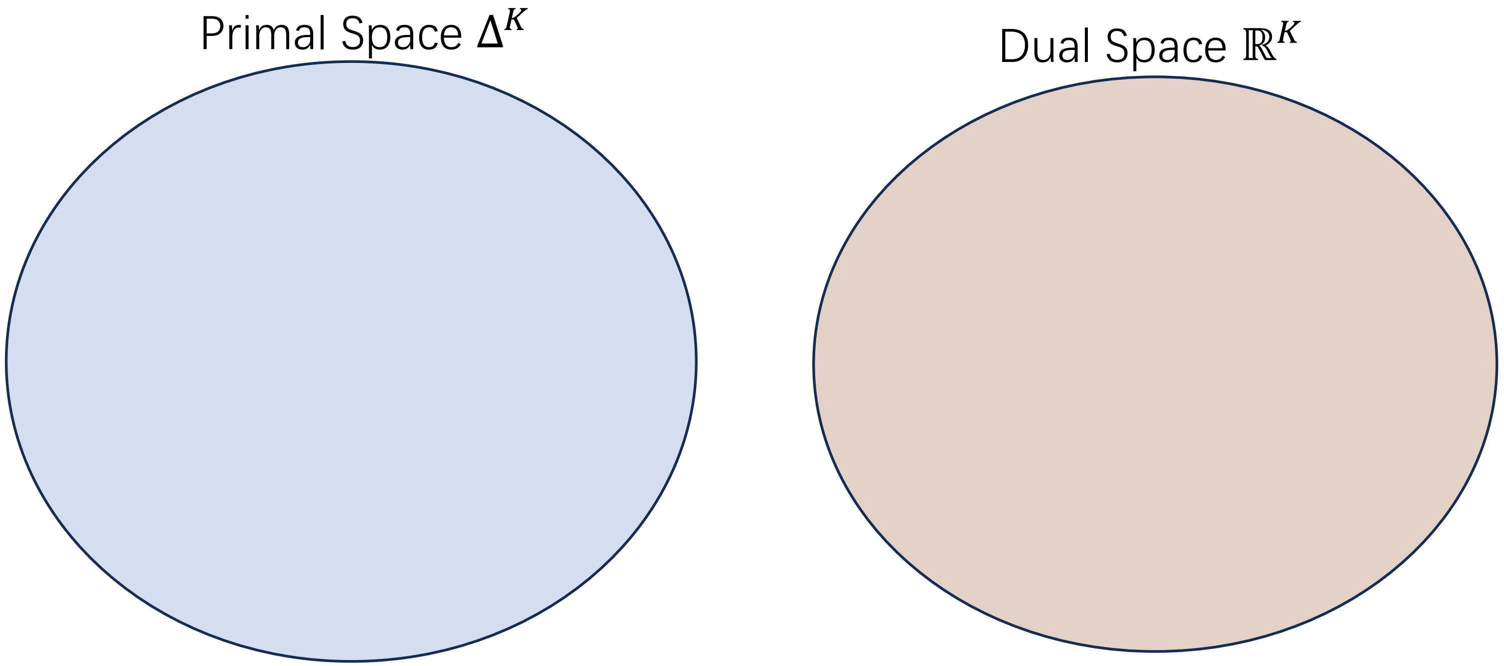

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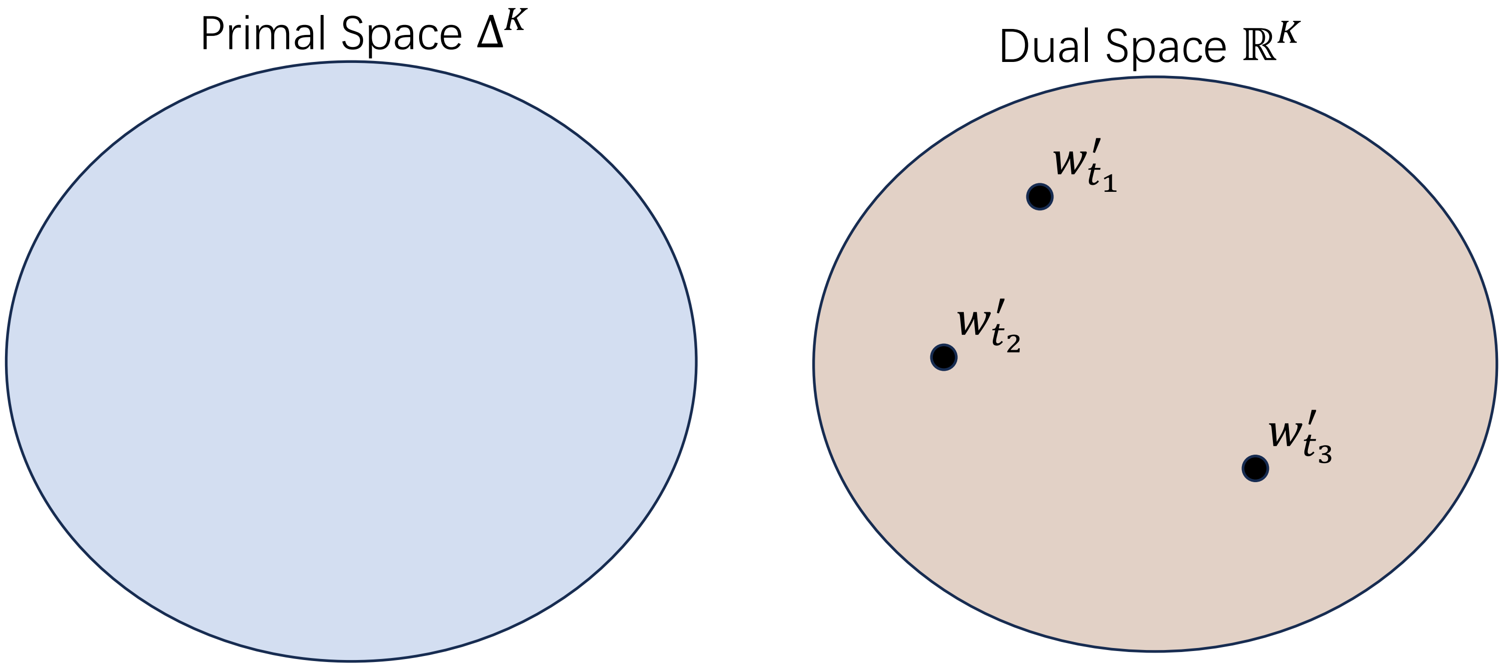

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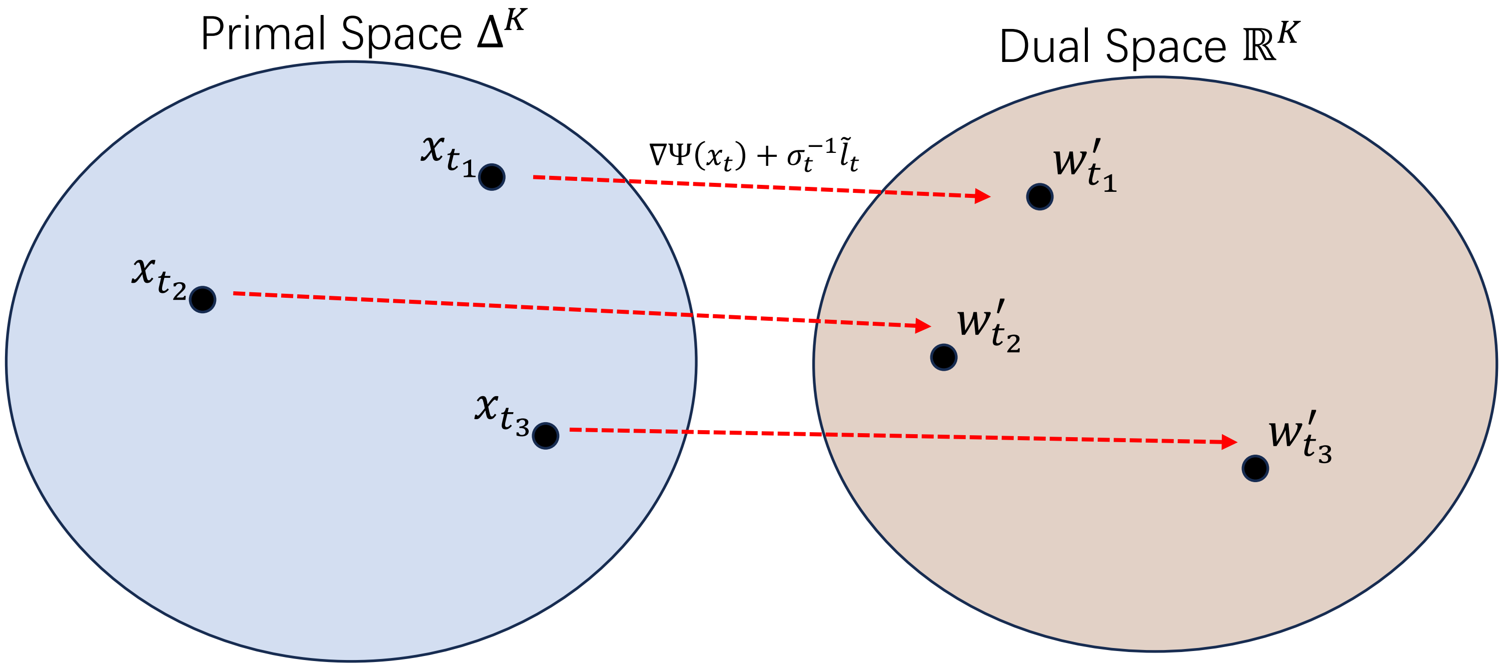

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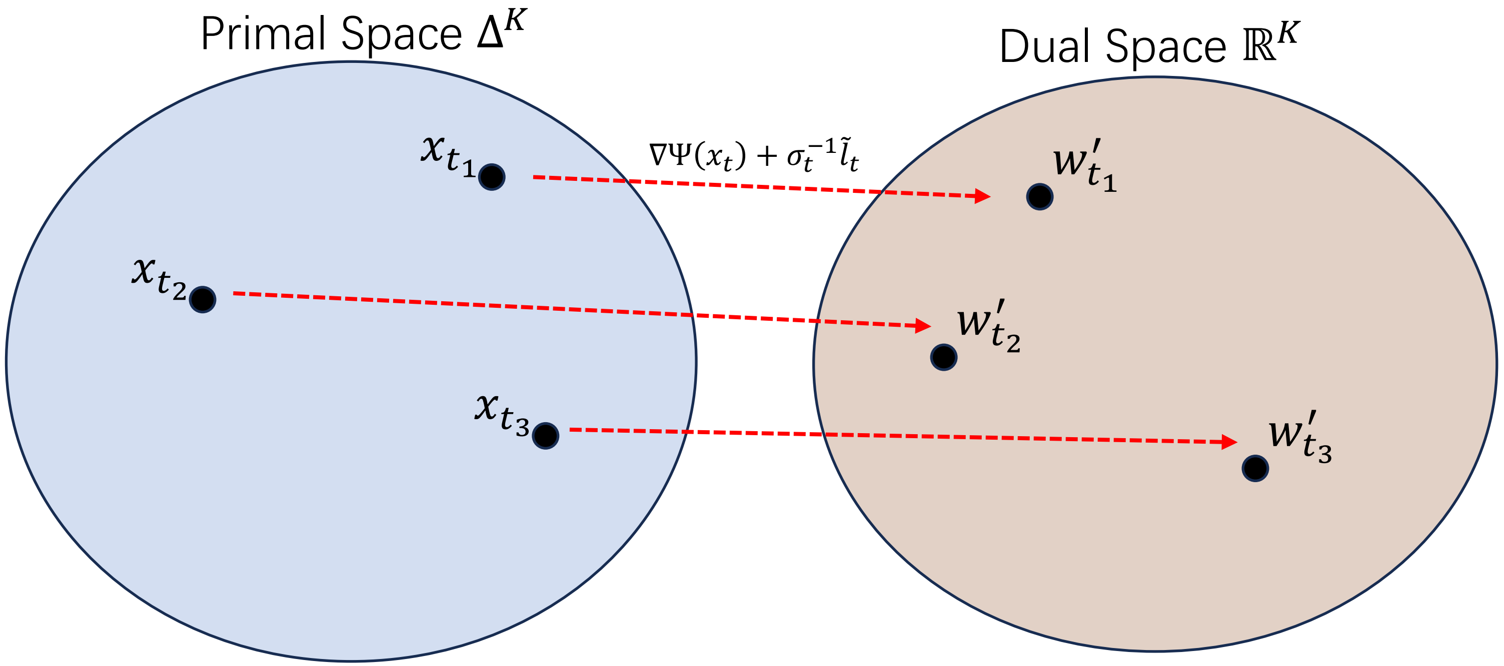

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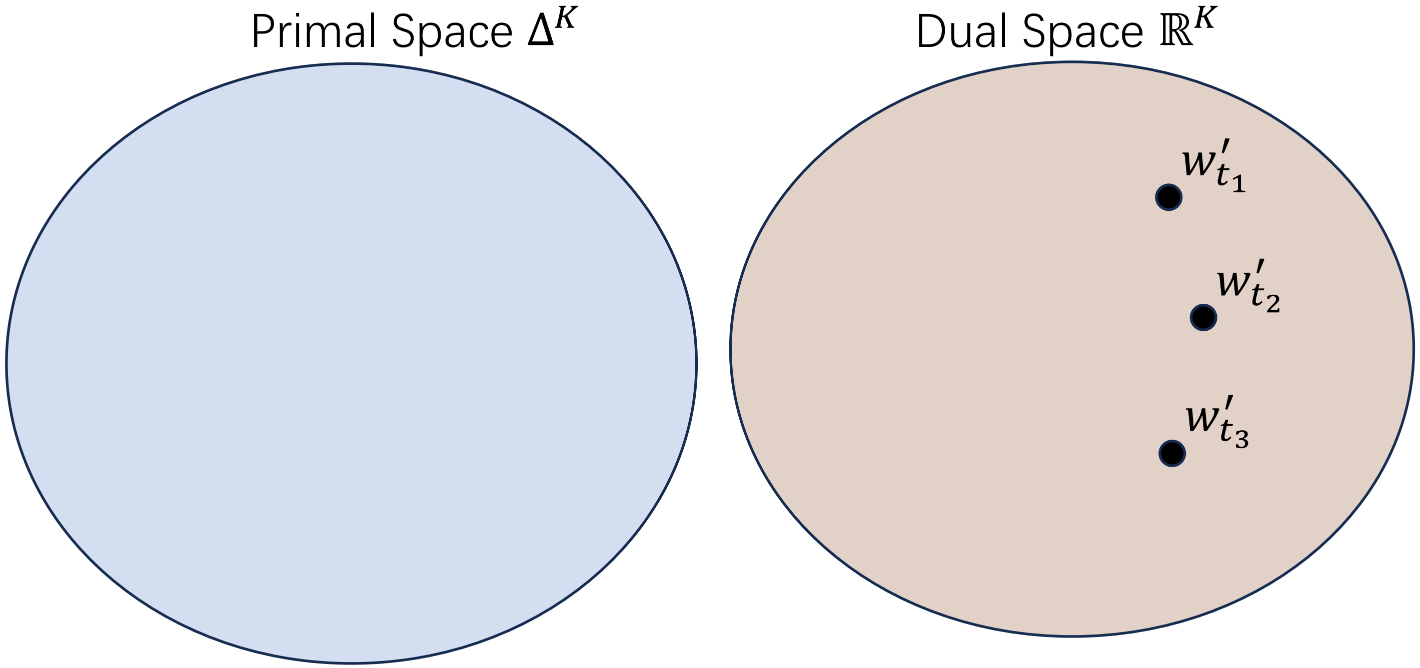

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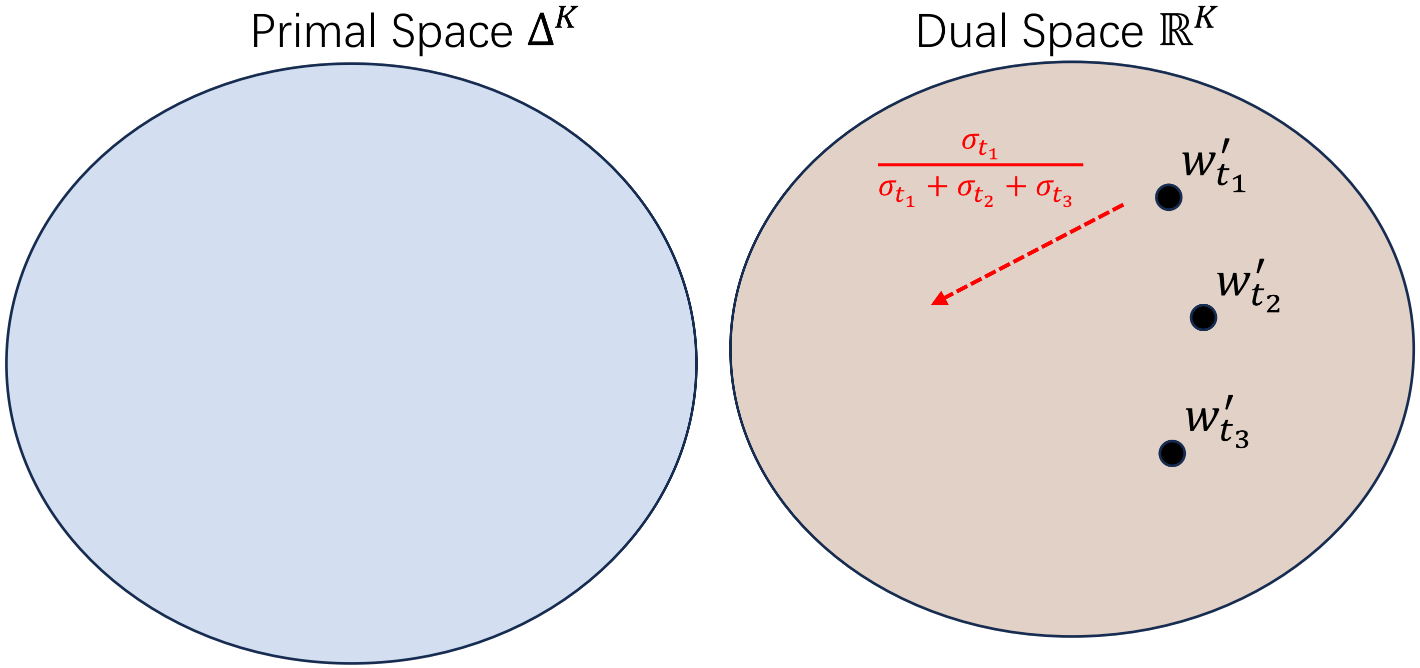

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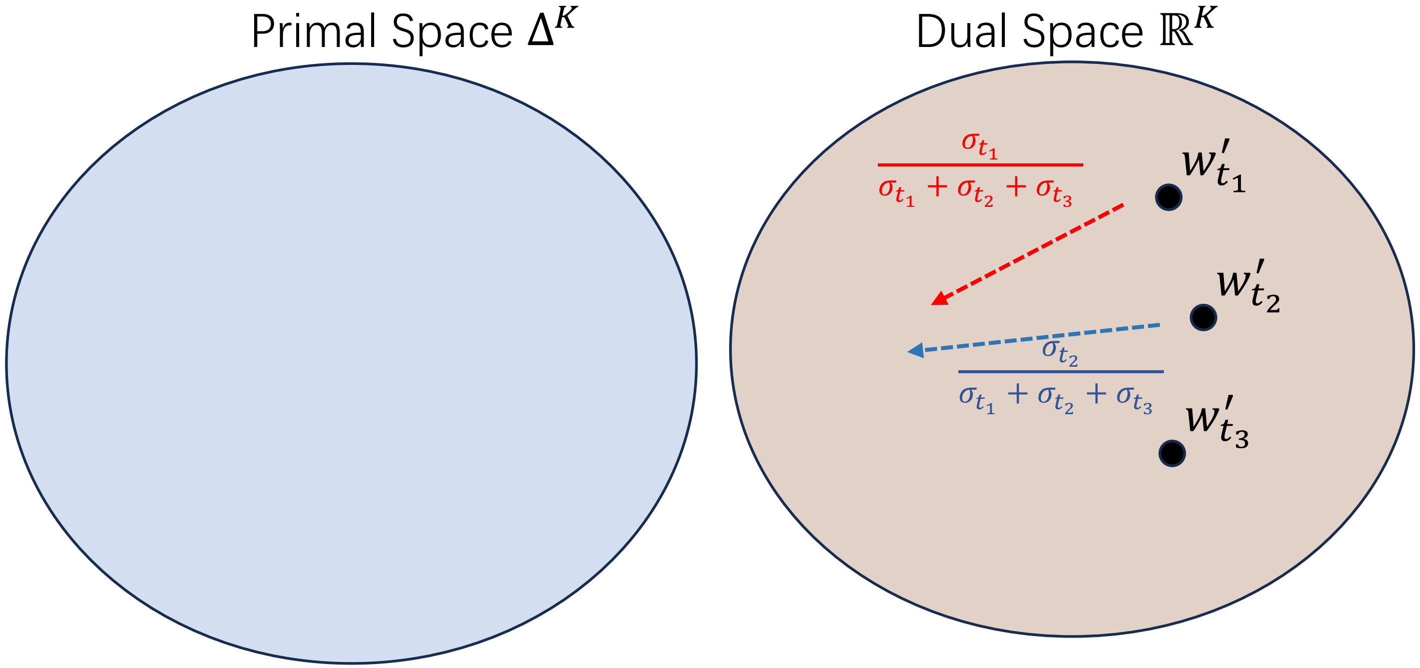

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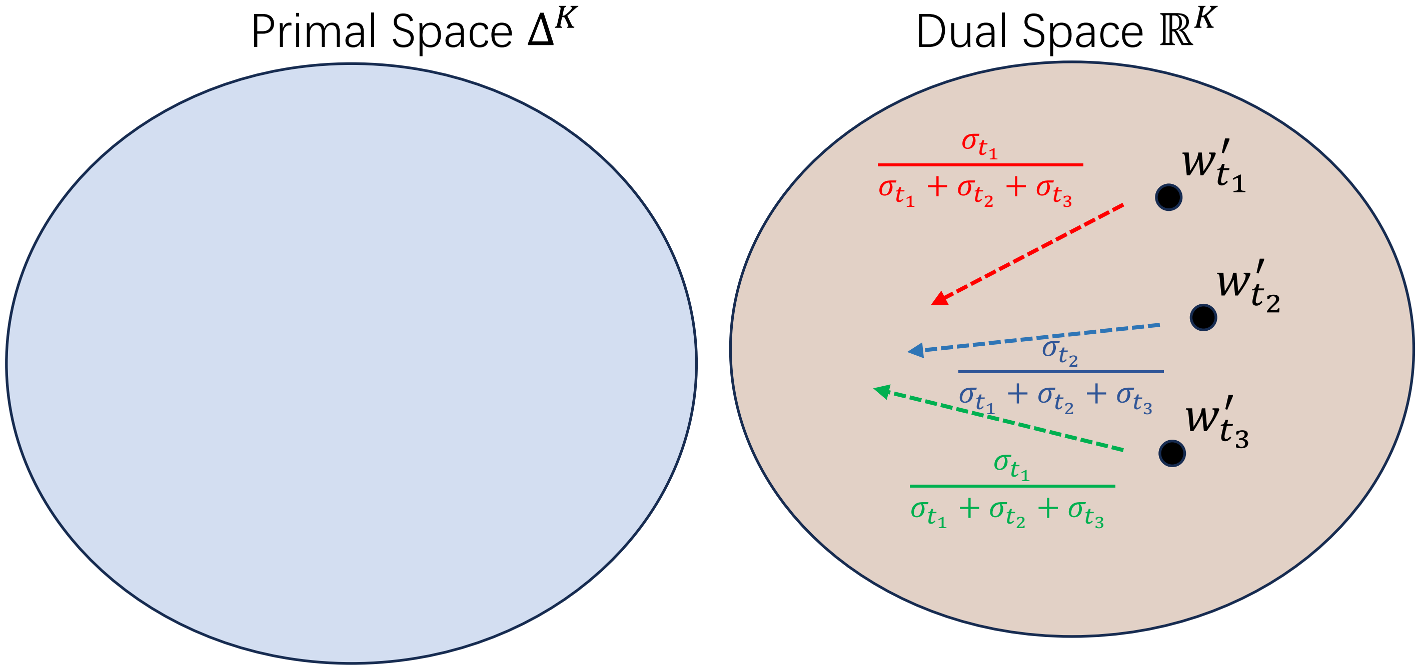

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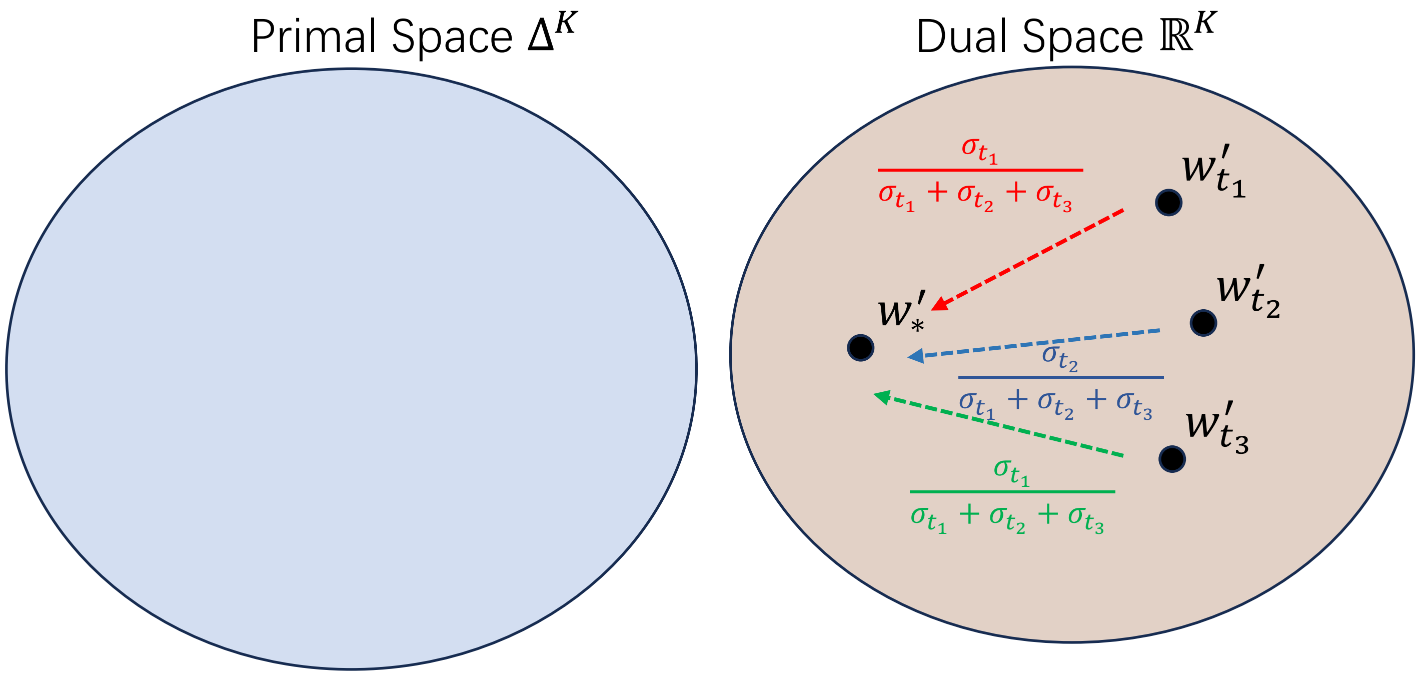

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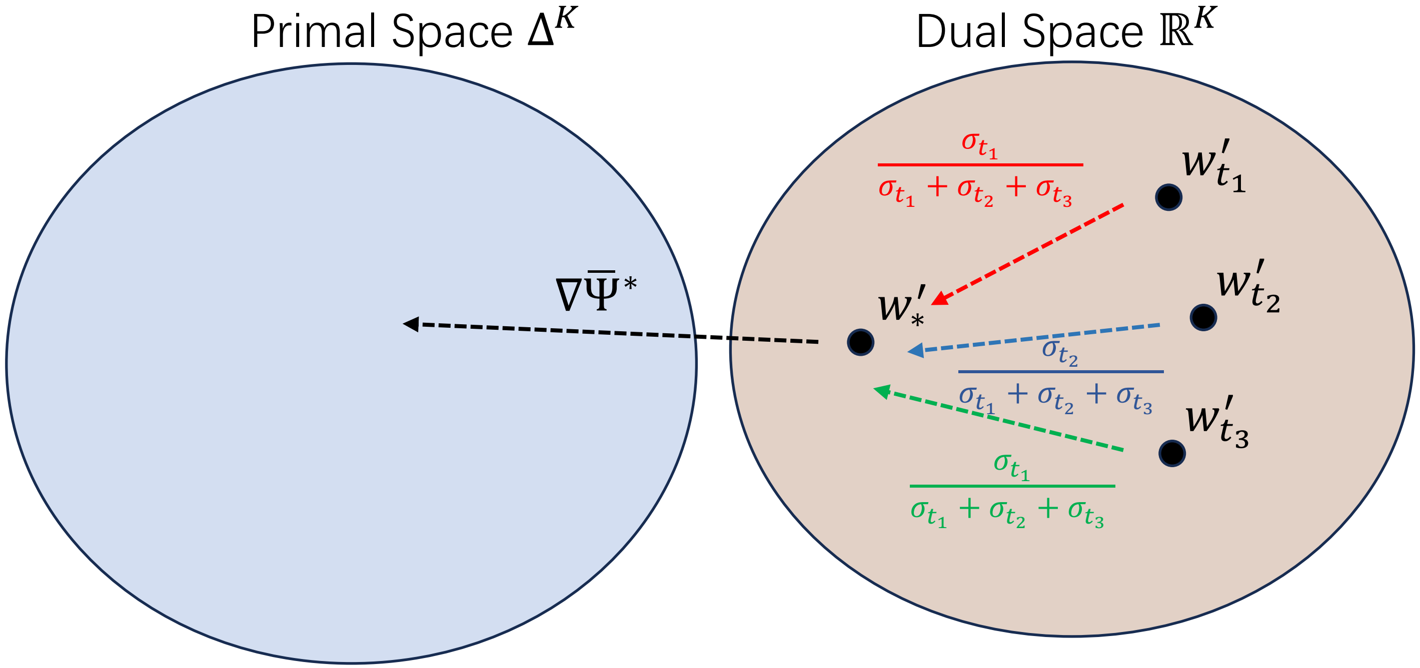

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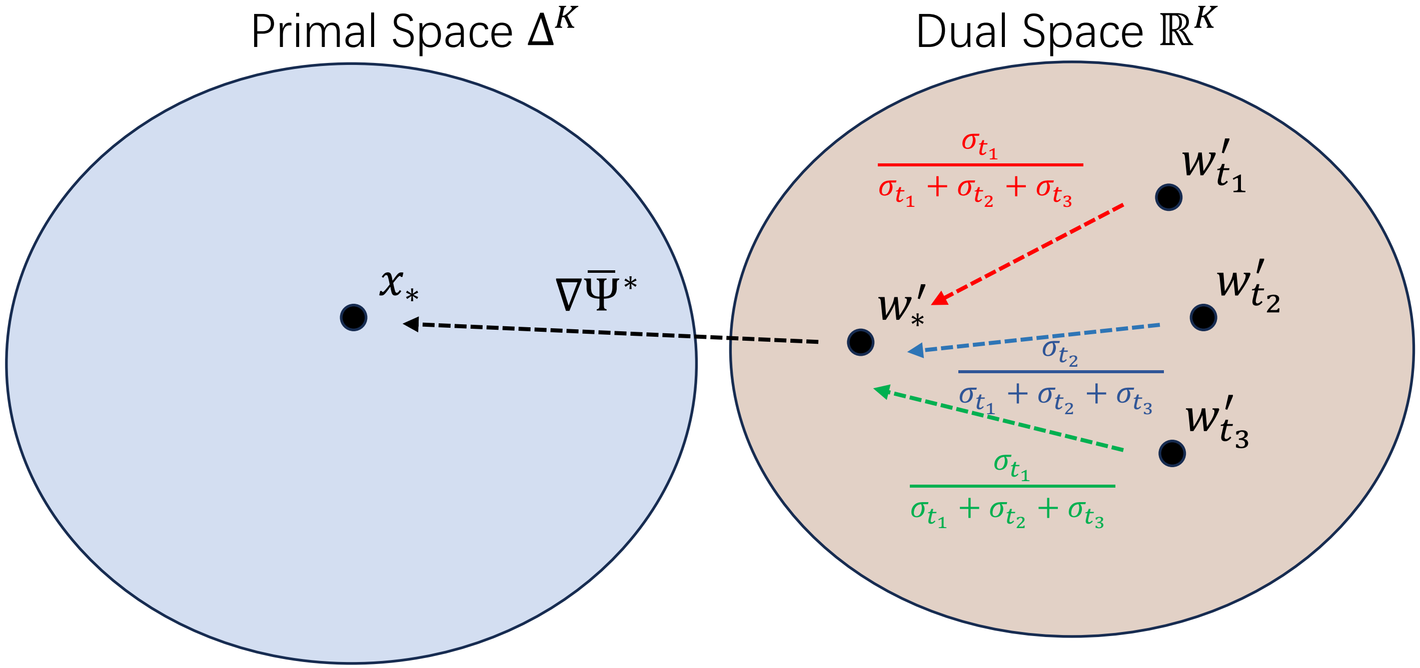

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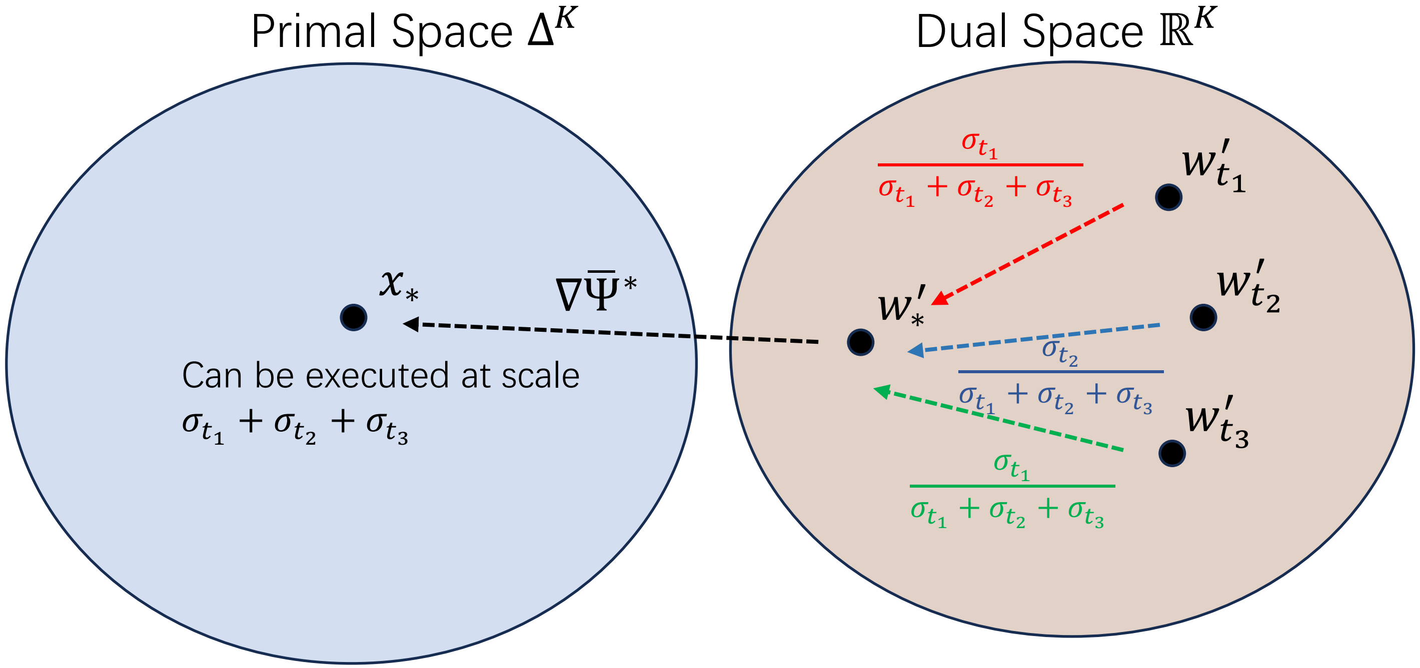

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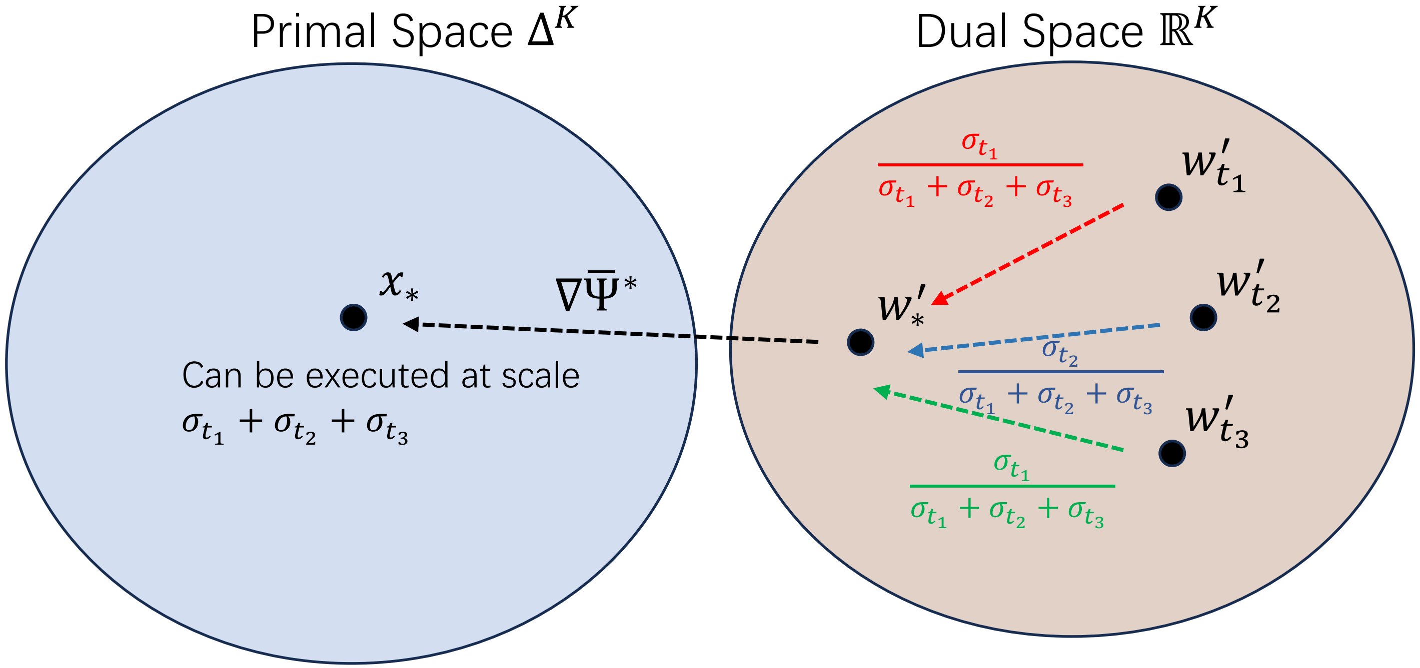
  - We are allowed to execute x^* at scale σ_Σ "**free of charge**"!



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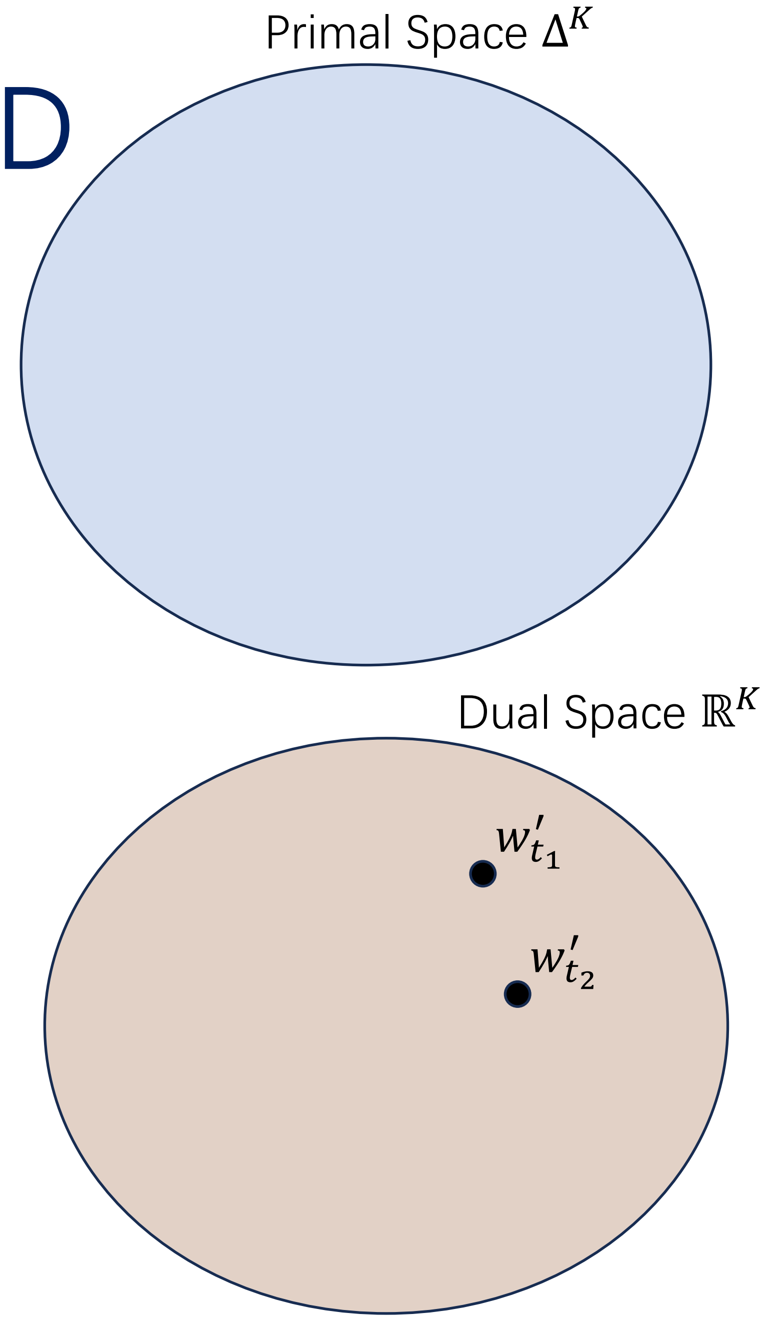

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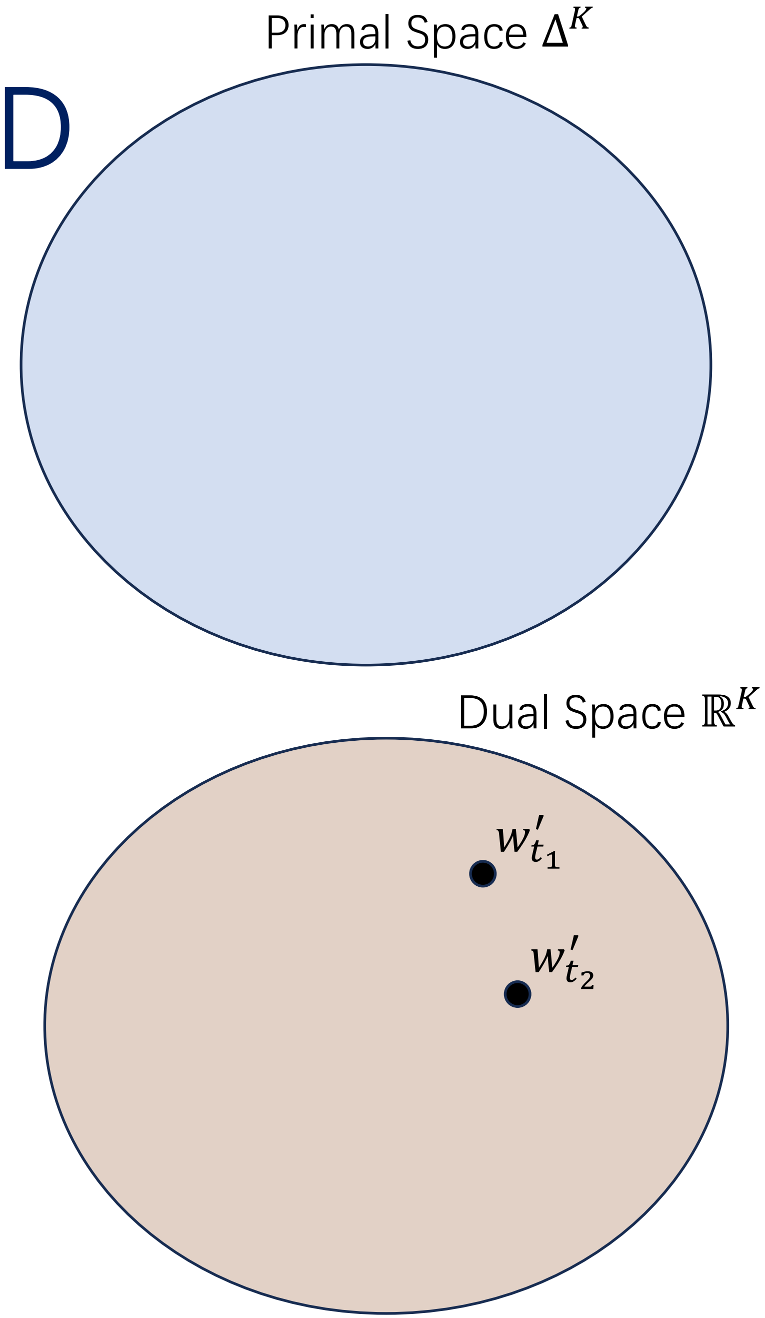
- Overdrafting:
  - Want if we want larger scale $\sigma_t > \sigma_{\sum} = \sigma_{t_1} + \sigma_{t_2}$?



High-Level Ideas of Banker-OMD

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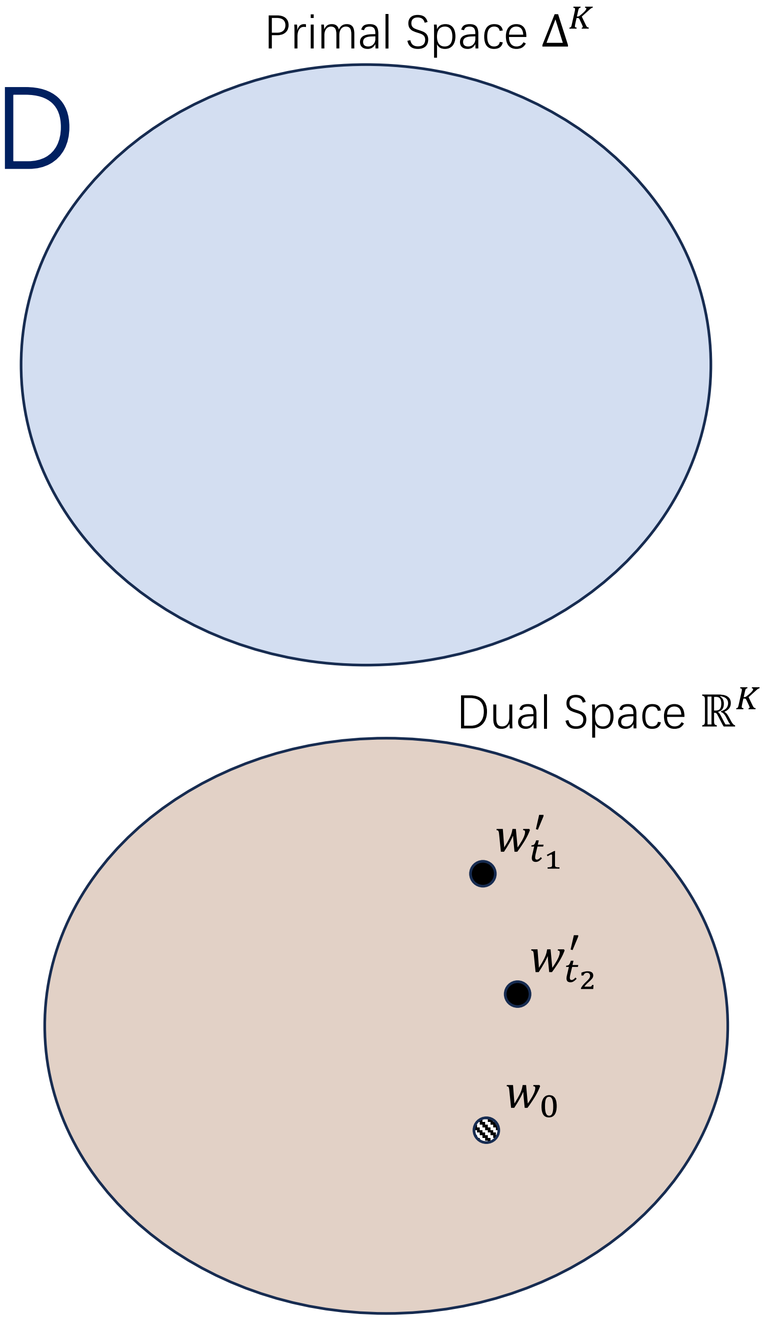

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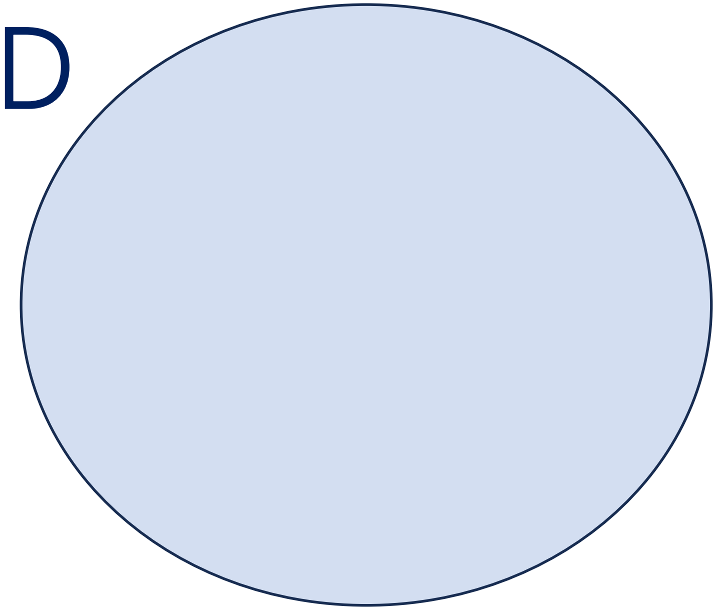
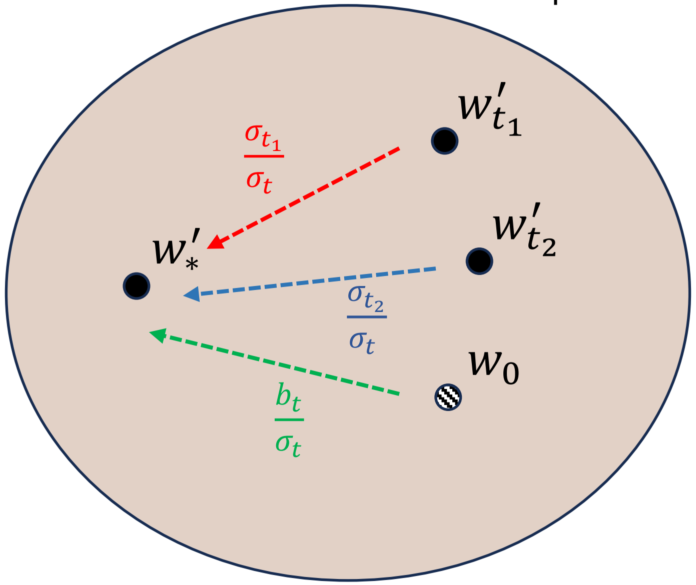
Primal Space Δ^K



Dual Space $\mathbb{R}^K$



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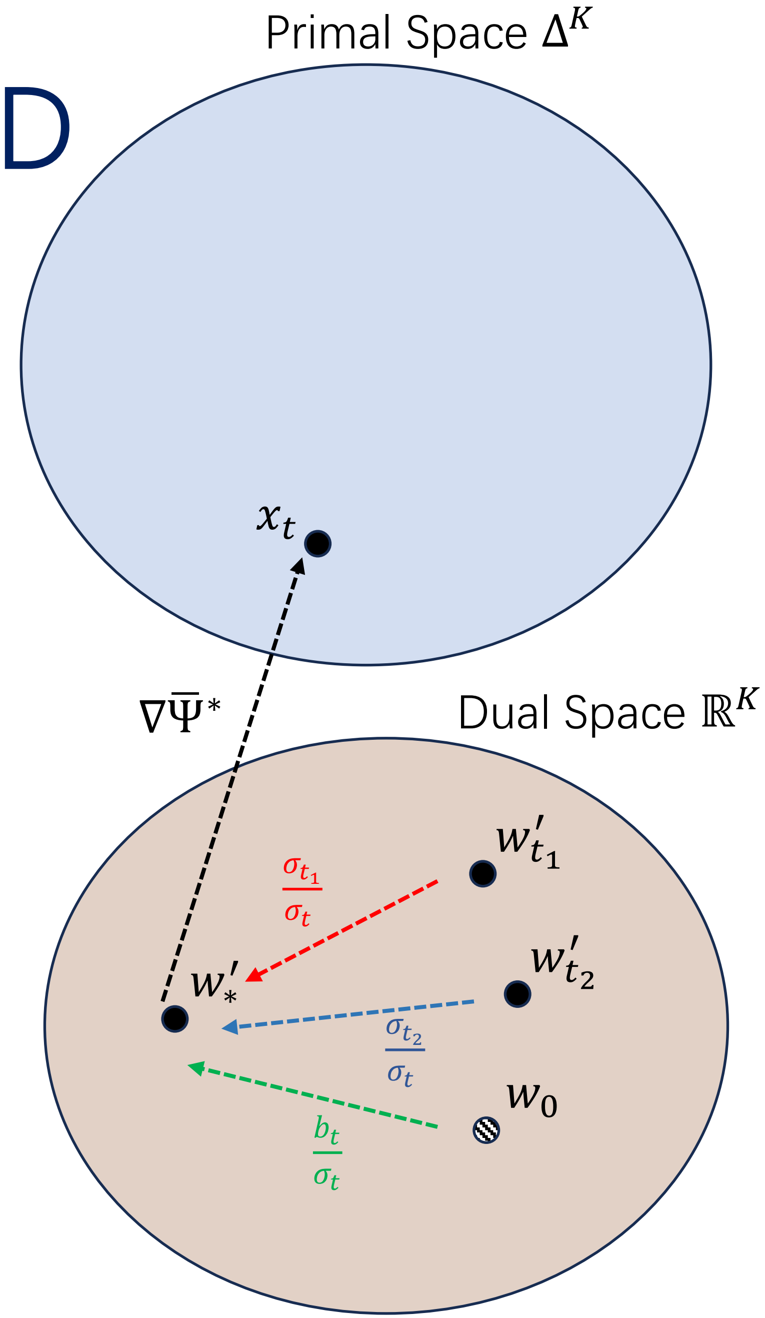

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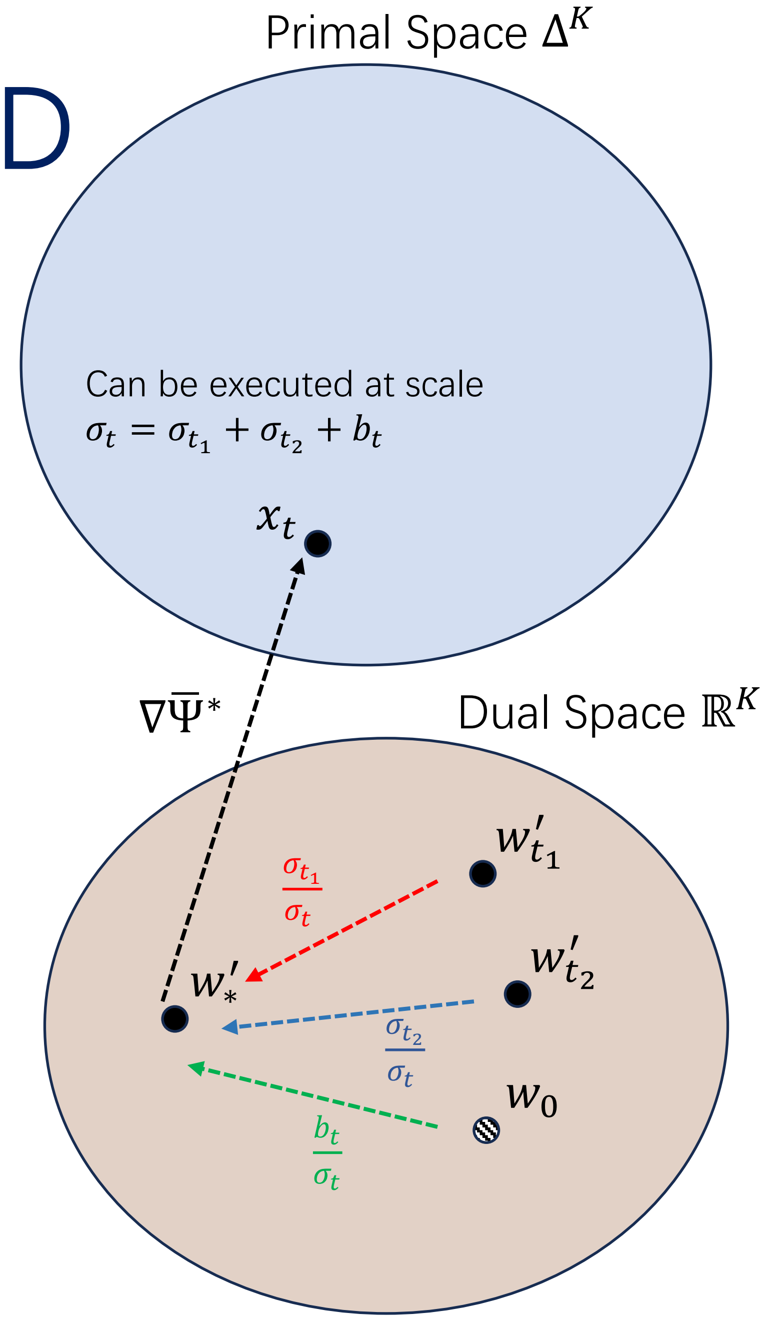

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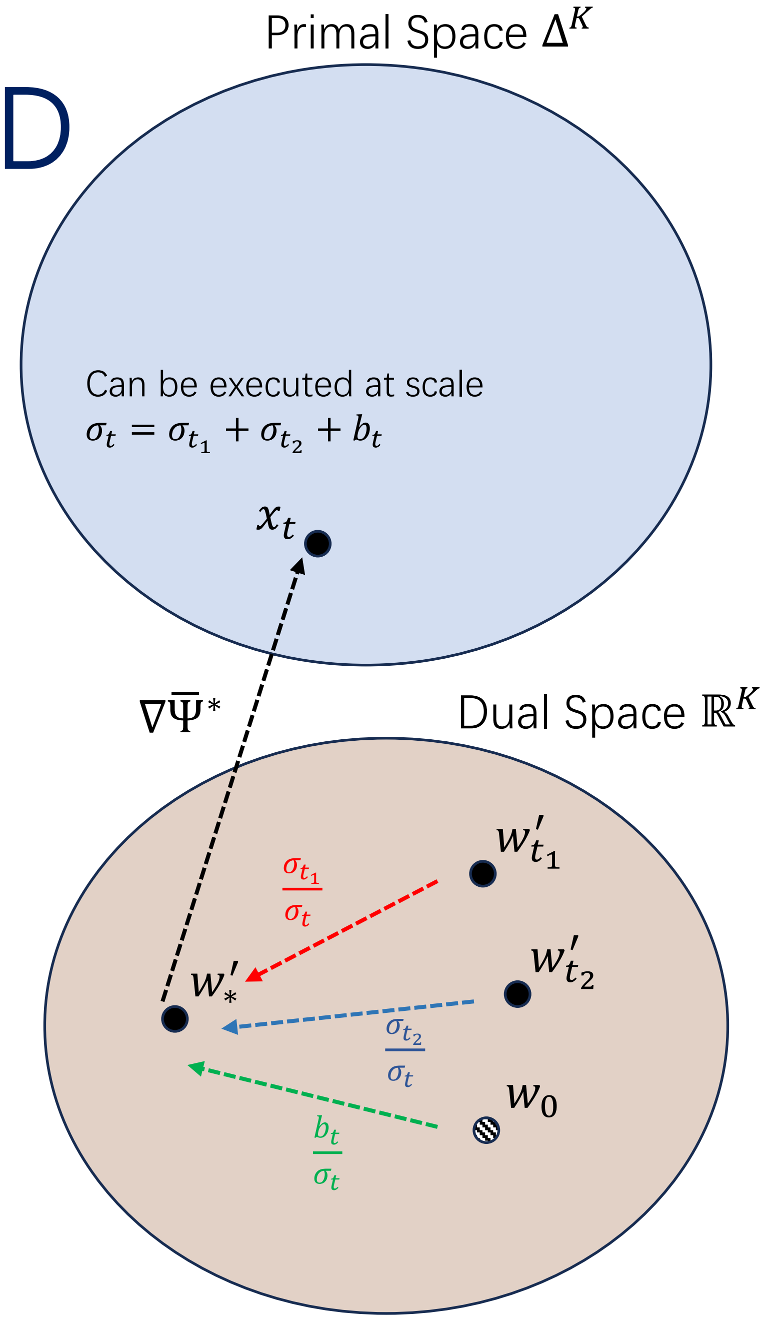

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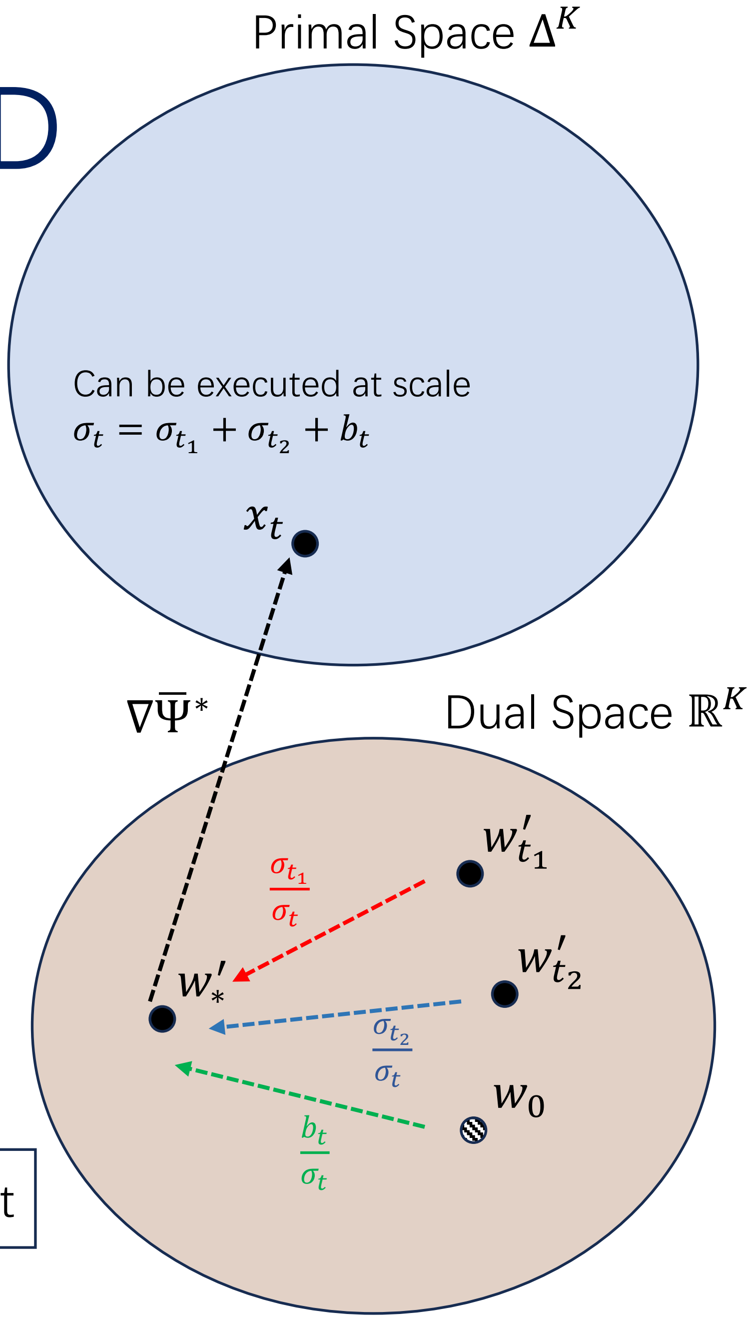

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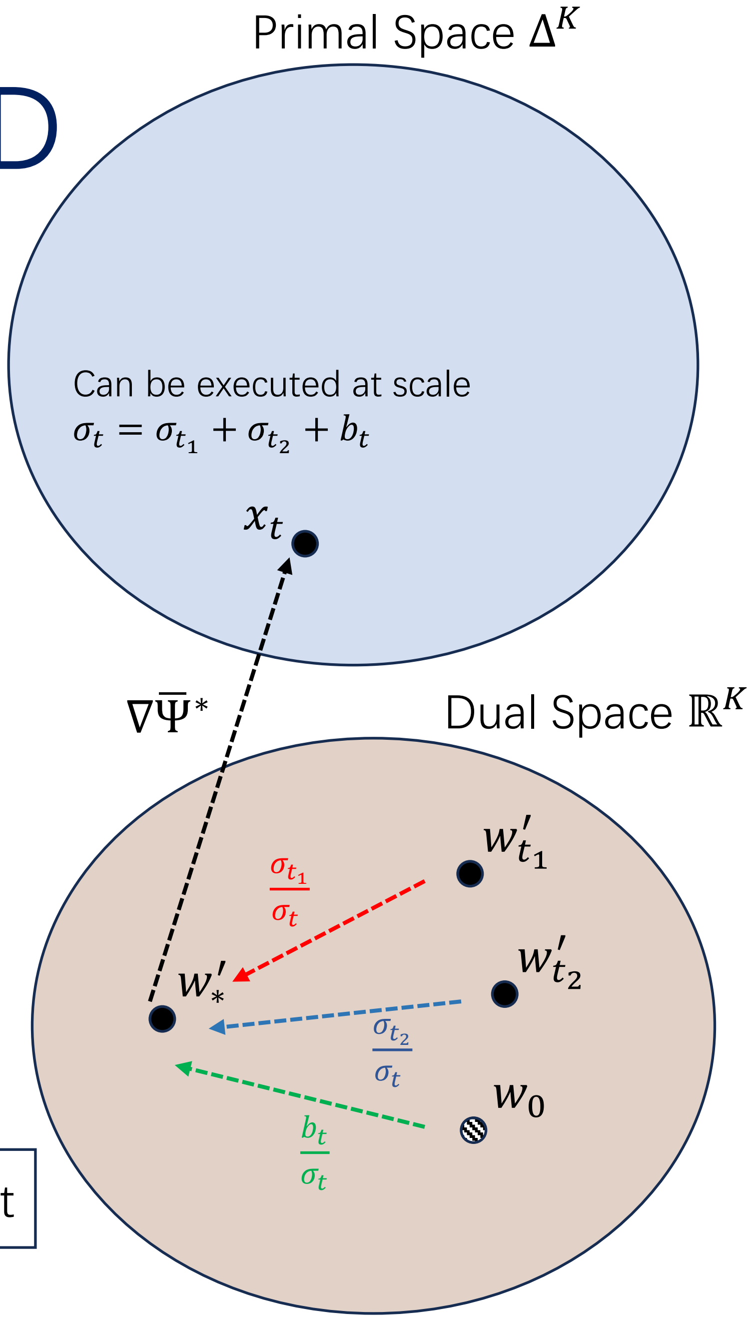

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  - Consistent rule for regret bookkeeping, ensuring

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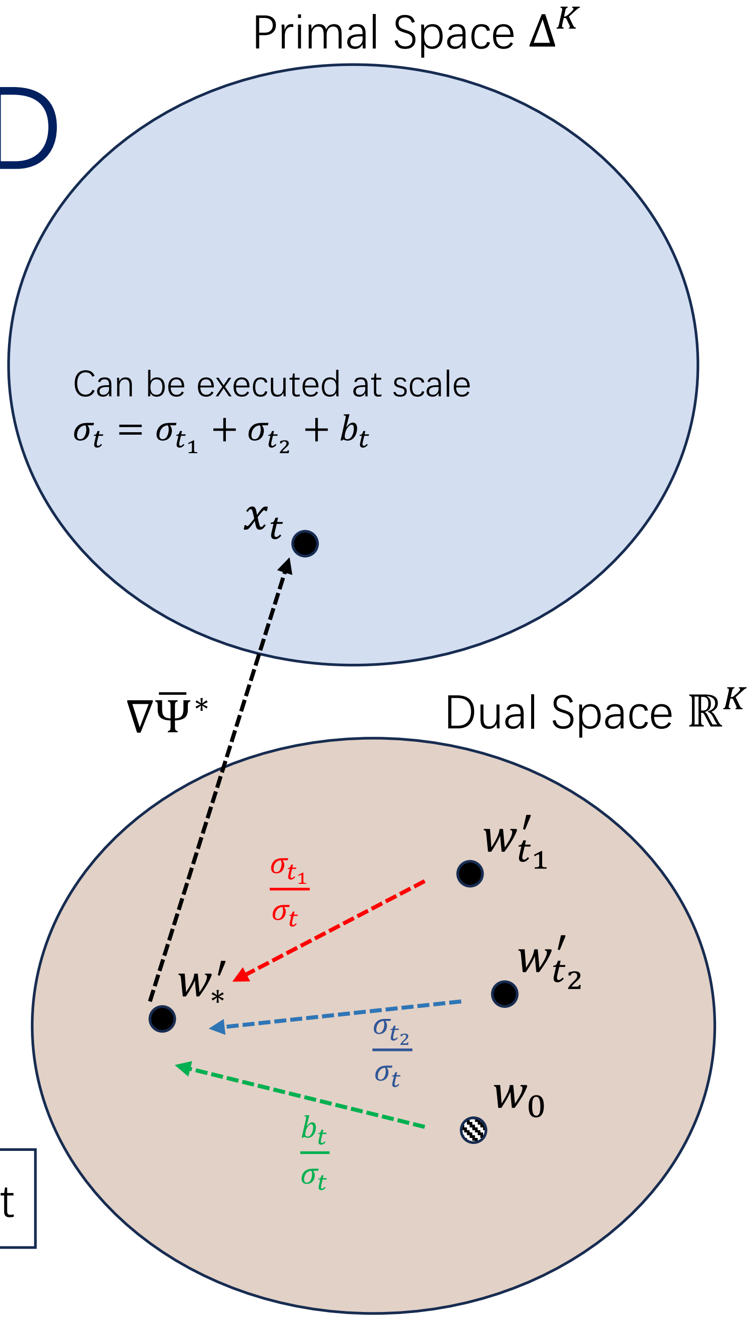

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  - And... provides general scale rule to deal with delays!

$$\tilde{\mathcal{O}}\left(\sqrt{D+T}\right) - \text{style bounds made easy!}$$

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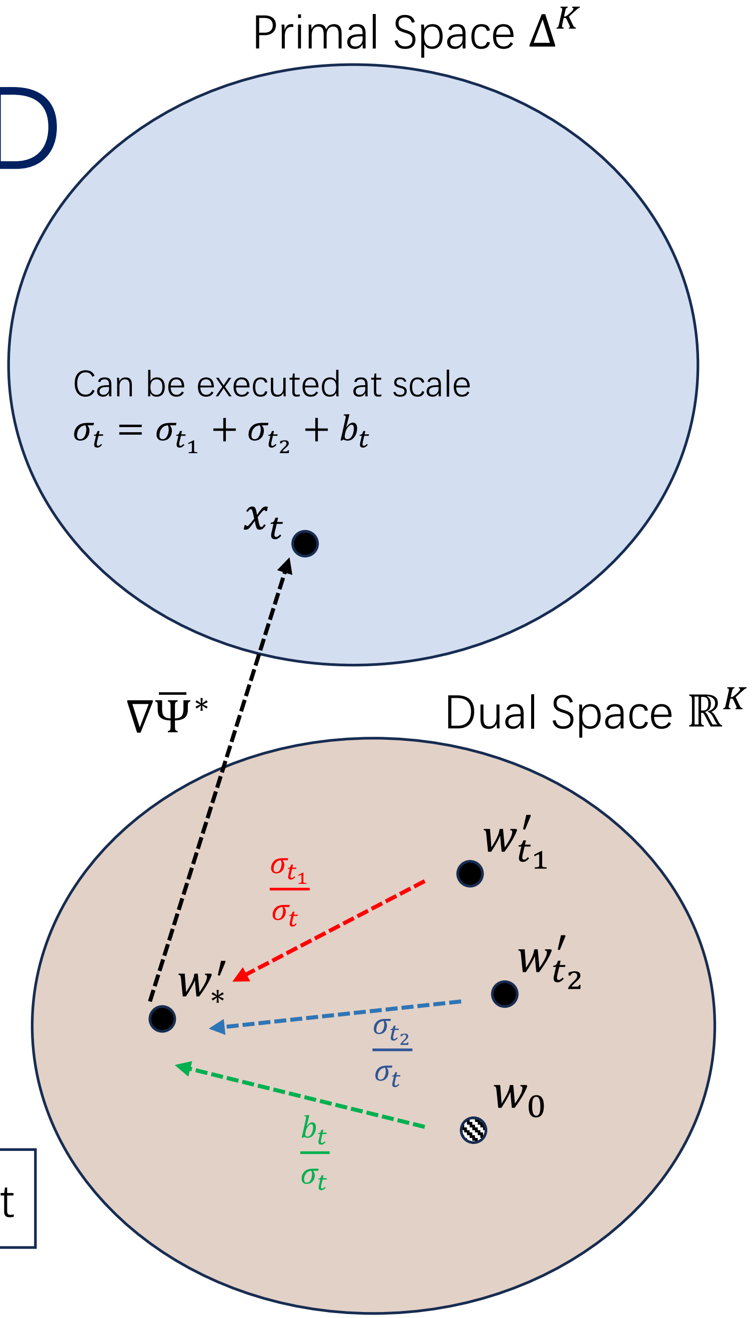
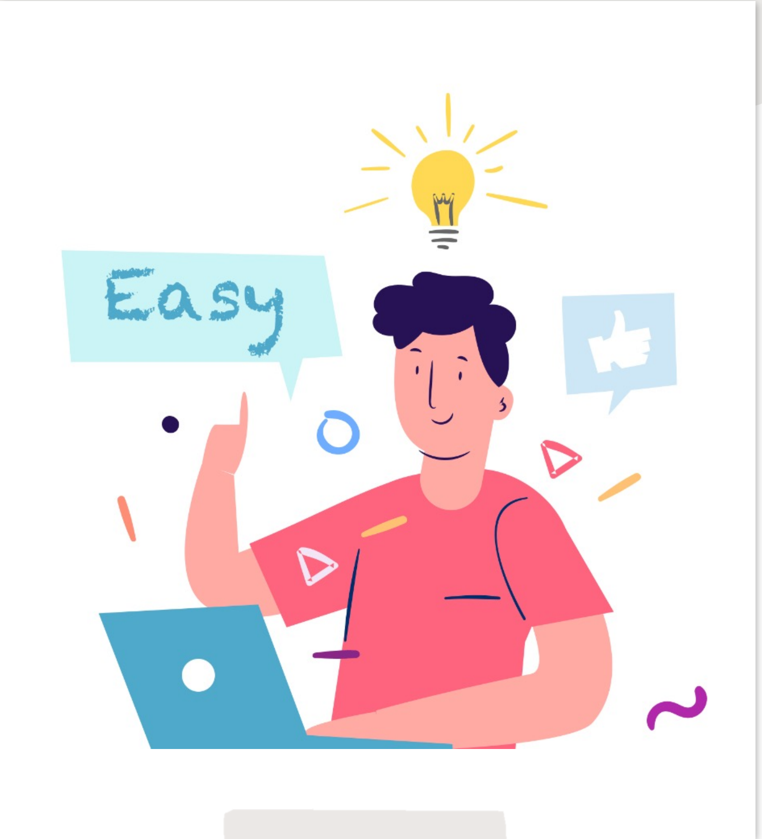


Main Theorem of Banker-OMD



- Given a practical algorithm based on vanilla OMD with $\mathcal{O}(C\sqrt{T})$ regret for non-delayed adversarial bandit problem, there is a Banker-OMD based version using the same regularizer, guaranteeing

$$\mathcal{O}\left(C\sqrt{T} + C'\sqrt{D\log D}\right)$$

regret in the delayed-feedback setting.

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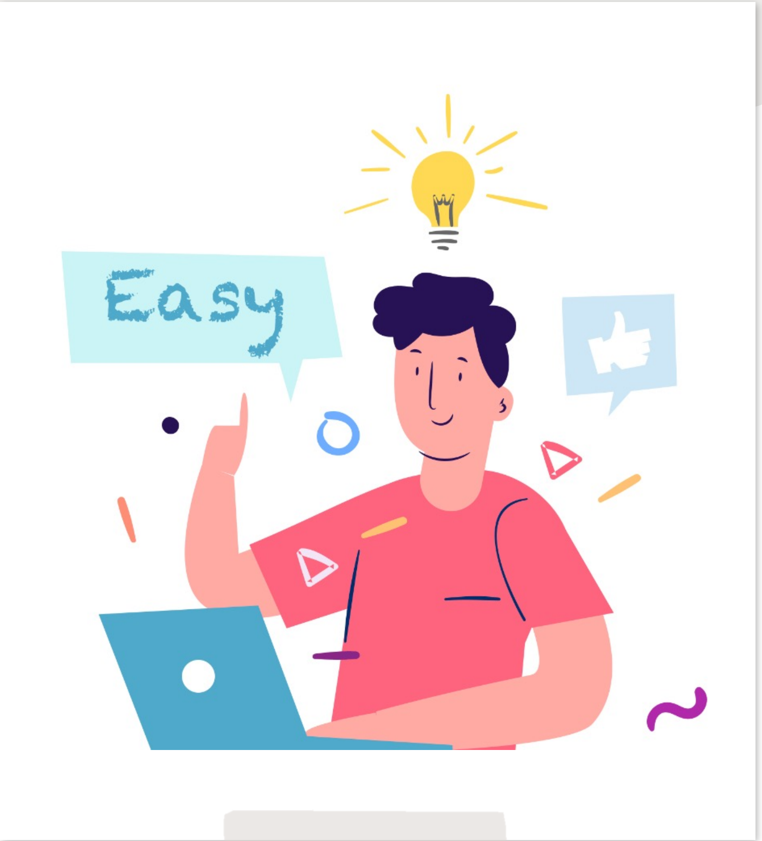

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- **Non-delayed** Algorithm ≈ **OMD** + Regularizer + Step-sizes
- **Delay-robust** Algorithm ≈ **Banker-OMD**+ Same regularizer + Modified step-sizes

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$$\tilde{O}\left(\sqrt{K(D+T)L}\right).$$

The End

- Thank for listening!

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