清華大學 交叉信息研究院
Institute for Interdisciplinary Information Sciences, Tsinghua University

PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE & ENGINEERING

Variance-Aware Sparse Linear Bandits

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Presented by Yan Dai

Problem Setup & Related Work

- A T-round game between an *agent* and an *environment* with **heteroscedastic noises**

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Main Theorem (informal). For **any** *variance-aware linear bandit algorithm* $\mathcal{A}$ whose regret $\mathfrak{R}_T^{\mathcal{A}}$ satisfies

$$\mathfrak{R}_T^{\mathcal{A}} = \tilde{O}\left(f(d)\sqrt{\sum \sigma_t^2} + g(d)\right) \text{ for some functions } f, g,$$

our framework gives a *variance-aware sparse linear bandit algorithm* $\mathcal{B}$ whose regret $\mathfrak{R}_T^{\mathcal{B}}$ satisfies

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Kim et al. (2022)	**Linear Bandit** (i.e., $s = d$) with varying σ_t	$\tilde{O}\left(d^{1.5}\sqrt{\sum_{t=1}^T \sigma_t^2} + d^2\right)$

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Thank You for Listening!

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References

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